Limited Asset Market Participation and Monetary Policy in a Small Open Economy

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Abstract

Limited asset market participation (LAMP) and trade openness are crucial features that characterize all real-world economies. We study equilibrium determinacy and optimal monetary policy in a model of a small open economy with LAMP. With low enough participation in asset markets, conventional wisdom concerning the stabilizing benefits of policy inertia can be overturned, irrespective of the constraint of a zero lower bound on the nominal interest rate. In contrast to recent studies, trade openness plays an important stabilizing role in LAMP economies. Optimal monetary policy is derived as a robust timeless rule, where the optimal level of inertia depends on the degree of trade openness. The optimal rule is shown to be super-inertial for standard economies, whereas the degree of inertia is significantly lower and not super-inertial for LAMP economies.

JEL: E31; E44; E52; E58; E63; F41

Keywords: limited asset market participation; small open economy; inverted aggregate demand logic; equilibrium determinacy; policy inertia; optimal monetary policy

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1 Introduction

Limited asset market participation (LAMP) is a well documented feature of both developing and developed economies. While its implications for monetary policy have recently been studied, the focus has largely been limited to closed economies. This paper seeks to address this gap. Our results suggest that trade openness and LAMP have important consequences for the design of monetary policy. First, we challenge the conventional wisdom on the benefits of policy inertia in monetary policy rules for the prevention of indeterminacy and self-fulfilling expectations. Second, we show that LAMP alters the trade-offs faced by a welfare-maximizing policymaker, such that super-inertial policy is no longer optimal in the presence of LAMP. In standard economies, we find that the optimal rule places a weaker response to both domestic inflation and output, and a lower degree of super-inertia, as the economy becomes more open to trade. However, in LAMP economies, the optimal policy coefficients become more strongly negative as trade openness increases, with a lower (possibly negative) weight for the degree of interest-rate smoothing.

LAMP is commonly introduced into two-agent New Keynesian (TANK) models by allowing for a share of 'rule-of-thumb consumers', a concept coined by Mankiw (2000) and further popularized by Galí et al. (2004).[1] Often referred to as 'hand-to-mouth consumers' (e.g, by Kaplan et al., 2014), these households differ from Ricardian consumers in that they hold no assets and consume all current income. The empirical evidence supports the inclusion of a large share of hand-to-mouth behaviour. For example, Aguiar et al. (2020) estimate that 40% of US households are hand-to-mouth based on the Panel Study of Income Dynamics. For low and middle-income countries, financial exclusion is estimated to be significantly higher, as illustrated by Figure 1.

This paper makes two main contributions to the literature. First, we examine the determinacy properties of a small open economy (SOE) with LAMP, focusing on the role of monetary policy inertia and trade openness for indeterminacy of popular Taylor-type feedback rules with and without the zero lower bound on the nominal interest rate. Then, we replace the feedback rule with a microfounded welfare criterion and examine the implications of LAMP and trade openness for optimal monetary policy under commitment, where the central bank is concerned about nominal interest rate volatility.

[1] It has been shown that for many purposes TANK models provide an appropriate theoretical shortcut to fully heterogeneous-agent New Keynesian models. See, e.g., Debortoli and Galí (2017) and Bilbiie (2020).

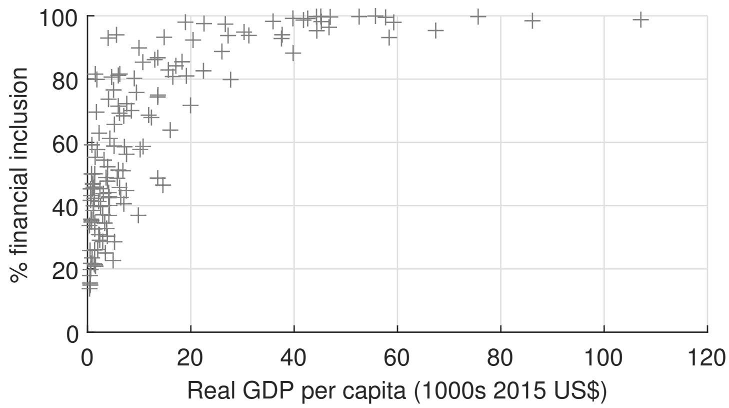


Figure 1: Financial inclusion against real GDP per capita. Financial inclusion is defined as the % of (aged 15+) population with an account at a financial institution or with a mobile-money-service provider, source: World Bank, Global Financial Inclusion Database. Real per capita GDP, source: World Bank national accounts data, and OECD National Accounts data files.

1.1 Monetary Policy Inertia and Trade Openness

As shown by Bilbiie (2008), LAMP can overturn the contractionary aggregate demand effect of a real interest rate increase in a closed economy. This results in an 'inverted aggregate demand logic' (IADL) that requires an 'inverted Taylor principle' for determinacy. The emergence of IADL depends on whether the profit channel overshadows the labour income channel. In the absence of full asset market participation, the increase in firm profits via a fall in marginal cost can dominate the effect of lower wages, leading to an expansionary effect of increasing interest rates.[2] While Boerma (2014) and Buffie and Zanna (2018) examine the determinacy implications of LAMP in the open economy, they limit attention only to simple inflation-targeting rules.[3] We add to this literature by focusing on the role of monetary policy inertia, a well-documented feature of central bank behaviour, including price-level targeting rules (so-called Wicksellian rules).

We find that interest-rate smoothing has contrasting effects on the determinacy properties of standard and IADL economies. In the standard case, monetary policy inertia reduces the possibility of indeterminacy, whereas policy inertia increases its likelihood under IADL.

[2]Colciago (2011) and Ascari et al. (2017) argue that nominal wage stickiness dampens the profit channel and can help restore the Taylor principle in closed economies. Buffie (2013) shows, however, that real wage rigidity is key for preventing the emergence of IADL.

[3]Boerma (2014) considers interest-rate rules that react to domestic-price inflation and output, whereas Buffie and Zanna (2018) examine feedback rules that only target consumer-price inflation.

This highlights an important caveat concerning the benefits of relying on interest-rate smoothing to help alleviate potential problems of indeterminacy. In the absence of LAMP, Woodford (2003*a*, chap. 4), Bullard and Mitra (2007), and Ascari and Ropele (2009) find that indeterminacy can in general be eliminated by adopting super-inertial rules, while several studies show that price-level targeting in particular helps improve stability compared to inflation targeting.[4] More recently, these benefits have been described in terms of "make-up" strategies for central banks (see, e.g., Powell, 2020; Svensson, 2020).[5] We find that while determinacy is always possible under a price-level targeting rule in the standard case, there are many degrees of LAMP for which determinacy is not possible under IADL. These results are shown to be robust to a variety of popular specifications for the interest-rate rule, including the choice of inflation target and a policy response to output, and generalize to model versions that incorporate capital and investment spending, positive trend inflation, incomplete asset markets, and dominant (or local) currency pricing.

Trade openness is also found to have contrasting effects on determinacy in the standard and IADL economies, although this depends on both the degree of LAMP and inertia. While trade openness typically induces indeterminacy in the standard case, the opposite holds when the degree of LAMP is sufficiently high. We find that under price-level targeting, closed IADL economies are more prone to indeterminacy than open IADL economies.

In an extension of Bilbiie (2008), Boerma (2014) and Buffie and Zanna (2018) show that the inverted Taylor principle is less likely to hold in open economies because of the terms of trade channel of monetary policy, which exerts contractionary pressure after a rise in the real interest rate. In contrast, we find the benefits of openness in restoring the Taylor principle are undermined by interest-rate inertia. The scope for active policy is limited due to a lower bound on the inflation response coefficient, which becomes very large with even a small amount of inertia. While a policy response to output can help for some degrees of LAMP, this requires the central bank placing a large weight on output stabilization.

Following the analysis of the linear model, we also examine determinacy in the presence of

[4]See, e.g., Carlstrom and Fuerst (2002), Woodford (2003*a*, chap. 4), Vestin (2006), Gaspar et al. (2007), Dib et al. (2013), Giannoni (2014), Bernanke (2017), and McKnight (2018). As Holden (2022) shows, these benefits extend to a zero lower bound setting.

[5]Under such strategies policymakers seek to redress past deviations of inflation from its target. Assuming a make-up rule enjoys credibility, undershooting (overshooting) the target will raise (lower) inflation expectations, lower (raise) the real interest rate and help to stabilize the economy. Inertial Taylor rules have by design the make-up feature as they commit to a response of the nominal interest rate to a weighted average of past inflation with the weights increasing with the degree of interest-rate smoothing.

a zero lower bound (ZLB). We show that policy inertia and trade openness both increase indeterminacy under a ZLB, regardless of the degree of LAMP. Moreover, under a ZLB the determinate region that arises under the Taylor principle in IADL economies is found to be extremely unstable. Overall, we find that policy inertia is detrimental to achieving determinacy under a ZLB in both standard and IADL economies, unless the policy rule also responds to a lagged 'shadow rate'.

1.2 Optimal Monetary Policy

Our second contribution is to extend the optimal monetary policy analysis of Bilbiie (2008) to the open economy dimension with interest-rate inertia. We derive optimal monetary policy under an equitable allocation using government transfers.[6] Similar to Woodford (2003*a*, chap. 6); Giannoni and Woodford (2003) and Levine, McAdam and Pearlman (2008), we allow for the costs of interest rate volatility to enter the loss function of the policymaker. Under commitment, the implicit instrument rule is shown to be robustly optimal and timeless. Optimal policy is found to be super-inertial for standard economies, whereas the degree of inertia is significantly lower and not super-inertial for IADL economies. In both cases, the optimal level of inertia depends on the degree of trade openness.

There are two policy trade-offs that welfare-maximizing policymakers face. The first is between interest rate stability and the stability of domestic inflation and output. The second is the standard trade-off between inflation and output stabilization. We find that the penalty on interest rate variability only affects the degree of activeness of the optimal rule and not the degree of inertia. In standard economies, the optimal inflation targeting rule is shown to be super-inertial with positive coefficients for inflation and the output gap. All policy coefficients are increasing under LAMP and decreasing with trade openness. In contrast, the policy coefficients are negative for IADL economies and become more strongly negative, as LAMP decreases and trade openness increases. For empirically-plausible intervals of LAMP, the optimal degree of interest-rate smoothing is low, and can be negative if the economy becomes sufficiently open.

Our analysis contributes to a small literature that characterizes optimal monetary policy in the presence of financial-excluded households. For the closed economy, Bilbiie (2008) shows that the optimal monetary policy under commitment is robustly optimal, where the

[6] In doing so, our normative analysis relates to empirical studies which find that consumption inequality closely tracks income inequality (Aguiar and Bils, 2015).

optimal response to inflation is decreasing in the share of LAMP. For open economies, Iyer (2016) finds that when the degree of LAMP is high, the policymaker, in addition to domestic inflation, should also stabilize the nominal exchange rate by putting more weight on stabilizing output.[7] We extend the analysis of both Bilbiie (2008) and Iyer (2016) by deriving the optimal monetary policy for LAMP economies under interest-rate inertia.

1.3 Road-Map

The rest of the paper is structured as follows. Section 2 sets out the baseline SOE model with LAMP. Section 3 considers the issue of equilibrium determinacy both in the absence and presence of a ZLB. We also test the robustness of our results by relaxing several assumptions of the baseline model, such as introducing capital and investment spending, incomplete asset markets, positive trend inflation, and dominant currency pricing. Section 4 derives a welfare-theoretic social loss function focusing on the equitable allocation, before analyzing optimal monetary policy under commitment with inertia. Finally, Section 5 concludes. Detailed derivations and proofs are provided in an online appendix.

2 A Small Open Economy Model with LAMP

This section presents our theoretical setup. It nests both the influential representative-agent SOE framework of Galí and Monacelli (2005) and the closed-economy LAMP model of Bilbiie (2008). The economy is comprised of perfectly competitive wholesale firms that produce a final good and monopolistically competitive retailers that sell intermediate tradable goods under Calvo (1983) price setting. There are two types of households in the economy. In addition to standard Ricardian households, we include an exogenous fraction of constrained households that do not have access to asset markets.

2.1 Households

Households are divided into two types. A fraction of households, $\lambda \in (0, 1)$, participate in domestic and international financial markets; these are referred to as Ricardian households and are denoted by superscript R. The remaining households $1 - \lambda$, referred to as

[7] Lahiri et al. (2007) consider the implications of LAMP for the optimal exchange rate regime in a flexible-price SOE.

constrained households and denoted by superscript C, consume only out of wage income, and have no assets or access to financial markets.

For both household types, $i = \{C, R\}$, single-period utility is assumed to be:

$$\begin{aligned} U_t^i = U(C_t^i, N_t^i) &= \frac{\left(C_t^i\right)^{1-\sigma}}{1-\sigma} - \frac{\left(N_t^i\right)^{1+\varphi}}{1+\varphi} \text{ for } \sigma \neq 1 \\ &= \log\left(C_t^i\right) - \frac{\left(N_t^i\right)^{1+\varphi}}{1+\varphi}; \quad \text{as } \sigma \to 1 \end{aligned} \tag{2.1}$$

where C_t^i is real consumption by household type i, σ is the coefficient of relative risk aversion (CRRA), N_t^i is labour supply of type i, and φ is the inverse of the Frisch elasticity.[8]

2.1.1 Ricardian Households

Ricardian households solve an intertemporal consumption problem:

$$\max_{C_t^R, N_t^R} \mathbb{E}_t \left[\sum_{s=0}^{\infty} \beta^s U(C_{t+s}^R, N_{t+s}^R) \right] \tag{2.2}$$

subject to a sequence of nominal budget constraints given by:

$$P_t^B B_{H,t} + P_t^{B^*} \mathcal{E}_t B_{F,t}^* = B_{H,t-1} + \mathcal{E}_t B_{F,t-1}^* + P_t W_t N_t^R - P_t C_t^R + \Gamma_t. \tag{2.3}$$

$B_{H,t}$ and $B_{F,t}^*$ are domestic and foreign bonds, denominated in the respective currencies, bought at the nominal price $P_t^B = 1/R_t$ and $P_t^{B^*} = 1/R_t^*$, where R_t and R_t^* denote the domestic and foreign nominal interest rate, respectively. P_t is the consumer price index (CPI) and $\mathcal{E}_t$ is the nominal exchange rate, measured as the domestic price of a unit of foreign currency. Finally, W_t and Γ_t denote the real wage rate and nominal profits, respectively.

Maximizing (2.2) subject to the budget constraint we obtain:

$$P_t^B = \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}^R}{\Pi_{t,t+1}} \right], \tag{2.4}$$

[8] If $\sigma \to 1$ the functional form is consistent with a balanced growth path concept of the steady state.

$$P_t^{B*} = \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}^R}{\Pi_{t,t+1}} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right], \tag{2.5}$$

$$\frac{U_{N,t}^R}{U_{C,t}^R} = -\left(C_t^R\right)^{\sigma} \left(N_t^R\right)^{\varphi} = -W_t, \tag{2.6}$$

where $\Pi_{t,t+1} \equiv \frac{P_{t+1}}{P_t}$ denotes the CPI inflation rate and $\Lambda_{t,t+1}^R \equiv \beta \frac{U_{C,t+1}^R}{U_{C,t}^R}$ is the stochastic discount factor for Ricardian consumers.

2.1.2 Consumption Demand

Households demand consumption goods from domestic H and foreign F retailers (imports):

$$C_t = \left[w_C^{\frac{1}{\mu_C}} C_{H,t}^{\frac{\mu_C - 1}{\mu_C}} + (1 - w_C)^{\frac{1}{\mu_C}} C_{F,t}^{\frac{\mu_C - 1}{\mu_C}} \right]^{\frac{\mu_C}{\mu_C - 1}}. \tag{2.7}$$

The weight w_C in the consumption basket attached to domestic consumption demand is a measure of home bias (where $w_C = 1$ is the autarky case). Maximizing total consumption (2.7) subject to a given aggregate expenditure $P_t C_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t}$ yields:

$$C_{H,t} = w_C \left(\frac{P_{H,t}}{P_t} \right)^{-\mu_C} C_t, \tag{2.8}$$

$$C_{F,t} = (1 - w_C) \left(\frac{P_{F,t}}{P_t} \right)^{-\mu_C} C_t. \tag{2.9}$$

Substituting these demand schedules into (2.7) gives the corresponding price index:

$$P_t = \left[w_C (P_{H,t})^{1-\mu_C} + (1 - w_C)(P_{F,t})^{1-\mu_C} \right]^{\frac{1}{1-\mu_C}}, \tag{2.10}$$

Foreign aggregate consumption C_t^* is given by an exogenous process. The real exchange rate is defined as the relative aggregate consumption price $Q_t \equiv \frac{P_t^* \mathcal{E}_t}{P_t}$. Then the foreign counterpart of the import demand schedule (2.9), which determines the export demand of the home good, is

$$C_{H,t}^* = (1 - w_C^*) \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\mu_C^*} C_t^* = (1 - w_C^*) \left(\frac{P_{H,t}}{P_t Q_t} \right)^{-\mu_C^*} C_t^*. \tag{2.11}$$

$P^*_{H,t}$ and P^*_t denote the respective prices of home-produced (i.e., imported) consumption goods and of aggregate consumption goods in the rest of the world (RoW) in foreign currency, and we have used the law of one price for differentiated goods, $\mathcal{E}_t P^*_{H,t} = P_{H,t}$. We impose perfect exchange rate pass-through for imports and because the home country is small, the law of one price implies that $P^*_t = P^*_{F,t}$, $\mathcal{E}_t P^*_t = P_{F,t}$, so $Q_t = \frac{P_{F,t}}{P_t}$. We can then write (2.11) as:

$$C^*_{H,t} = (1 - w^*_C) \left(\frac{1}{S_t} \right)^{-\mu^*_C} C^*_t, \tag{2.12}$$

where $S_t \equiv \frac{P_{F,t}}{P_{H,t}}$ are the terms of trade (ToT). Finally, total exports per capita is defined as $EX_t \equiv C^*_{H,t}$.

2.1.3 Constrained Consumers

Constrained consumers have no income from monopolistically competitive retail firms and must consume out of wage income. Their nominal consumption is given by:

$$P_t C^C_t = P_t W_t N^C_t. \tag{2.13}$$

Constrained consumers choose C^C_t and N^C_t to maximize an analogous utility function to (2.2) but subject to (2.13). The first order conditions can be written as:

$$\frac{U^C_{N,t}}{U^C_{C,t}} = -\left(C^C_t\right)^{\sigma} \left(N^C_t\right)^{\varphi} = \frac{U^R_{N,t}}{U^R_{C,t}} = -W_t, \tag{2.14}$$

which has the same form as eq. (2.6) for the Ricardian consumers, but as we shall discuss further below, C^C_t and N^C_t are not the same as C^R_t and N^R_t in general.

With both Ricardian and constrained households, aggregate consumption and hours supplied are given by:

$$C_t = \lambda C^R_t + (1 - \lambda) C^C_t, \tag{2.15}$$

$$N_t = \lambda N^R_t + (1 - \lambda) N^C_t. \tag{2.16}$$

2.2 Firms

There are wholesale and retail firms. The former act in perfect competition producing a homogeneous final good, whereas the latter produce and sell differentiated intermediate goods under monopolistic competition.

2.2.1 Wholesale Sector

Wholesale firms hire labour N_t to produce homogeneous output Y_t^W using the standard labour-augmenting constant returns to scale production technology:

$$Y_t^W = F(N_t, A_t) = A_t N_t. \tag{2.17}$$

Profit maximization implies:

$$P_t W_t = P_t^W F_{N,t} = P_t^W \frac{Y_t^W}{N_t} \quad \Rightarrow W_t = MC_t \left(\frac{P_{H,t}}{P_t} \right) \frac{Y_t^W}{N_t}, \tag{2.18}$$

where $MC_t \equiv \frac{P_t^W}{P_{H,t}}$ is real marginal cost in units of domestic retail output.

2.2.2 Retail Sector

A retail firm m converts an amount of wholesale output $Y_t^W(m)$ into a differentiated good of amount $Y_t(m) - F(m)$, where $F(m) = F$ are fixed costs assumed to be equal across retail firms. The retail differentiated goods are combined into the final good Y_t using a CES-aggregator production technology:

$$Y_t \equiv \left[\int_0^1 Y_t(m)^{\frac{\varsigma-1}{\varsigma}} dm \right]^{\frac{\varsigma}{\varsigma-1}}. \tag{2.19}$$

The CES technology implies demand schedules for each intermediate input j given by:

$$Y_t(m) = \left[\frac{P_{H,t}(m)}{P_{H,t}} \right]^{-\varsigma} Y_t. \tag{2.20}$$

Following Calvo (1983), in every period each retail firm m faces a fixed probability $1 - \xi$ of being able to optimally set their price to $P_{H,t}^0(m)$. If the price is not re-optimized, then it is held fixed. The objective of a retail producer m at time t is to choose $P^0_{H,t}(m)$ to maximize discounted real profits:

$$\mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \frac{\Lambda_{t,t+k}}{P_{t+k}} Y_{t+k}(m) \left[P^0_{H,t}(m) - P_{H,t+k} MC_{t+k} \right] \tag{2.21}$$

subject to

$$Y_{t+k}(m) = \left(\frac{P^0_{H,t}(m)}{P_{H,t+k}} \right)^{-\zeta} Y_{t+k}, \tag{2.22}$$

where $\Lambda_{t,t+k} \equiv \beta^k \frac{U_{C,t+k}}{U_{C,t}}$ is the stochastic discount factor over the interval $[t, t+k]$. This leads to the usual optimal price condition and aggregate law of motion for aggregate inflation:

$$\frac{P^0_{H,t}}{P_{H,t}} = \frac{\zeta}{(\zeta - 1)} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \Lambda_{t,t+k} \left(\Pi_{t,t+k}\right)^{-1} \left(\Pi_{H,t,t+k}\right)^{1+\zeta} Y_{t+k} MC_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \Lambda_{t,t+k} \left(\Pi_{t,t+k}\right)^{-1} \left(\Pi_{H,t,t+k}\right)^{\zeta} Y_{t+k}}, \tag{2.23}$$

$$1 = \xi \left(\Pi_{H,t-1,t}\right)^{\zeta-1} + (1-\xi) \left(\frac{P^0_{H,t}}{P_{H,t}} \right)^{1-\zeta}, \tag{2.24}$$

where $\Pi_{H,t-1,t} \equiv \frac{P_{H,t}}{P_{H,t-1}}$. The index m is dropped as all firms face the same marginal cost so the right-hand side of (2.23) is independent of firm size or price history. Aggregate output Y_t is given by:

$$Y_t = \frac{A_t N_t - F}{\Delta_t}, \tag{2.25}$$

where $\Delta_t \equiv \int_0^1 \left(\frac{P_{H,t}(m)}{P_{H,t}} \right)^{-\varsigma} dm \geq 1$ is the degree of price dispersion of retail goods which can be shown to follow the dynamic process:

$$\Delta_t = \xi \Pi^{\zeta}_{H,t-1,t} \Delta_{t-1} + (1-\xi) \left(\frac{P^0_{H,t}}{P_{H,t}} \right)^{-\zeta}. \tag{2.26}$$

2.3 Output Market Clearing

Output market clearing for retail firm m is:

$$Y_t(m) = C_{H,t}(m) + C^*_{H_t}(m).$$

Aggregating yields the following resource constraint:

$$Y_t = C_{H,t} + C^*_{H_t} = C_{H,t} + EX_t, \tag{2.27}$$

and using the demand conditions (2.8) and (2.12) yields:

$$Y_t = w_C \left(\frac{P_{H,t}}{P_t}\right)^{-\mu_C} C_t + (1 - w^*_C) \left(\frac{1}{S_t}\right)^{-\mu^*_C} C^*_t. \tag{2.28}$$

2.4 Monetary Policy

The nominal interest rate R_t is a policy variable given by the following explicit instrument (Taylor-type) rule:[9]

$$\log\left(\frac{R_t}{R}\right) = \rho_r \log\left(\frac{R_{t-1}}{R}\right) + \mathbb{E}_t\left[\theta_\pi \log\left(\frac{\Pi_{t,t+1}}{\Pi}\right) + \theta_y \log\left(\frac{Y_t}{Y}\right)\right], \tag{2.29}$$

where $\rho_r, \theta_\pi, \theta_y \geq 0$. We focus on forward-looking rules as many central banks target forecasted inflation in practice due to the observed time delay in the transmission mechanism of monetary policy.[10] In Section 4, we consider optimal targeting rules (i.e., implicit instrument rules) under commitment.

2.5 Foreign Bond Accumulation

In nominal terms and measured in the home country currency, foreign bond holdings evolve according to:

$$P_t^{B^*} \mathcal{E}_t B^*_{F,t} = \mathcal{E}_t B^*_{F,t-1} + P_t TB_t,$$

where the nominal trade balance $P_t TB_t = P_{H,t} Y_t - P_t C_t$ is the difference between domestic output and private consumption. Defining $B_{F,t} \equiv \frac{\mathcal{E}_t B^*_{F,t}}{P_t}$ to be the stock of foreign bonds in home country consumption units, it follows that

$$P_t^{B^*} B_{F,t} = \frac{\Pi^{\mathcal{E}}_{t-1,t}}{\Pi_{t-1,t}} B_{F,t-1} + TB_t, \tag{2.30}$$

[9]This is in 'implementable' form as proposed by Schmitt-Grohe and Uribe (2007). The conventional Taylor rule replaces the output level relative to its steady state $\frac{Y_t}{Y}$ with the output gap $\frac{Y_t}{Y_t^n}$ where Y_t^n is the natural rate, i.e., the level of output that would have prevailed if all prices were perfectly flexible.

[10]For further discussion, see Batini and Haldane (1999) and McKnight and Mihailov (2015).

where $\Pi^{\mathcal{E}}_{t-1,t} \equiv \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}$ is the (gross) nominal depreciation of the SOE currency.

2.6 Equilibrium

An equilibrium is defined in the model variables given the conditions outlined above together with the interest rate rule (2.29) and three structural exogenous shock processes A_t, C^*_t and R^*_t, which are assumed to follow stochastic AR(1) processes. Appendix A of the online appendix provides a summary of this equilibrium.

2.7 Symmetric Equilibrium of Small Open Economies with Risk Sharing

Up to now we have modelled the SOE in an environment consisting of the RoW, which from its own viewpoint is closed. For later use when we come to optimal policy, we now amend the environment to consist of a continuum of $i \in [0, 1]$ identical open economies of which the 'home' economy is just one. We assume there is international risk-sharing in this version of the model so the risk premium is zero. The first-order conditions (2.4) and (2.5) lead to the standard risk-sharing condition:

$$C^R_t = (C^R_t)^i Q^{\frac{1}{\sigma}}_{i,t} \tag{2.31}$$

where $Q_{i,t} \equiv \frac{\mathcal{E}_{i,t} P^i_t}{P_t}$ is the home country (or SOE) vis-à-vis country (or SOE) i bilateral real exchange rate, with $\mathcal{E}_{i,t}$ now the corresponding bilateral nominal exchange rate between these two countries (both identical SOEs). Naturally, the risk-sharing only applies to Ricardian and not constrained households.

Then using (2.8) and (2.12), in a symmetric equilibrium with $C_t = C^*_t$,[11] $\mu_C = \mu^*_C$, $\lambda_i = \lambda$, $\sigma = \sigma^*$ and $Q_t = \frac{\mathcal{E}_t P^*_t}{P_t}$ we have

$$\begin{aligned} Y_t &= Y^*_t = C_{H,t} + C^*_{H,t} \\ &= C_t \left(\frac{P_{H,t}}{P_t}\right)^{-\mu_C} \left(\mathrm{w}_C + (1 - \mathrm{w}_C) Q_t^{\mu_C - \frac{1}{\sigma}}\right) \end{aligned} \tag{2.32}$$

[11]Note that all macroeconomic quantities are in per capita form.

3 Stability and Determinacy Analysis

The model is linearized around a non-stochastic steady state where net inflation is zero, i.e., $\Pi = 1$, and prices $P = P_H = P_F = P^* = 1$. Then by definition the steady state terms of trade and real exchange rate are $\varepsilon = Q = 1$. For the LAMP aspects of the model, we follow Bilbiie (2008) and impose an equitable outcome $C^R = C^C$ and $N^R = N^C$, which can be achieved by assuming that free entry drives profits to zero in an equilibrium in the steady state with $\frac{F}{Y} = (1 - MC) = \frac{1}{\zeta}$.[12] Since the focus of this section is on (local) stability and equilibrium determinacy, we consider the deterministic perfect foresight case with all shocks set equal to zero. In what follows, all lower-case variables in this section denote percentage deviations from the steady state.

We can describe the non-policy aspects of the model using a New Keynesian Phillips Curve (NKPC) and an intertemporal IS curve, both expressed in terms of consumption by Ricardian consumers:[13]

$$\pi_{H,t} = \beta \pi_{H,t+1} + \Psi \Upsilon c_t^R, \tag{3.1}$$

$$c_t^R = c_{t+1}^R - \frac{w_C}{\sigma}\left(r_t - \pi_{H,t+1}\right), \tag{3.2}$$

$$y_t = \Xi c_t^R, \tag{3.3}$$

where the parameters are defined as:

$$\Upsilon \equiv \frac{\sigma(1-w_C)}{w_C} + \frac{\lambda(\varphi+\sigma)\left[w_C\varphi + \sigma\left(1+\frac{1}{\zeta}\right)\right] + \varphi(1-w_C)(\varphi+\sigma)\left[1+\frac{\omega\sigma}{w_C}\right]}{\lambda(\varphi+\sigma)\left(1+\frac{1}{\zeta}\right) - (1-\lambda)\varphi\left[w_C(1+\varphi) + (\sigma-1)\left(1+\frac{1}{\zeta}\right)\right]}, \tag{3.4}$$

$$\Xi \equiv w_C\lambda + (1-w_C)\left[1+\frac{\omega\sigma}{w_C}\right] + w_C(1-\lambda)\frac{1+\varphi}{\varphi+\sigma}\left(\Upsilon - \frac{\sigma}{w_C} + \sigma\right), \tag{3.5}$$

$\Psi \equiv \frac{(1-\xi)(1-\beta\xi)}{\xi} > 0$, and $\omega \equiv w_C(\mu_C - 1/\sigma) + \mu_C^* = \mu_C(1+w_C) - w_C/\sigma > 0$, if $\mu_C = \mu_C^*$.

The threshold for the proportion of Ricardian households λ below which the inverted

[12] As we discuss later in the paper, alternatively a subsidy scheme for the optimal equitable allocation in Proposition 5 in Section 4.2 can support this outcome.

[13] Alternative NKPC and IS expressions, written in terms of total consumption in deviations from baseline allocations, and hence in standard output gap terms, are discussed in Section 4.

aggregate demand logic (IADL) occurs is the point at which Υ changes sign. From (3.4) this is given by:

$$\lambda = \lambda^* = \frac{\varphi \left[w_C(1+\varphi) + (\sigma - 1)\left(1 + \frac{1}{\zeta}\right)\right]}{\varphi \left[w_C(1+\varphi) + (\sigma - 1)\left(1 + \frac{1}{\zeta}\right)\right] + (\varphi + \sigma)\left(1 + \frac{1}{\zeta}\right)}. \tag{3.6}$$

Then replacing $\lambda^* = \lambda^*(w_C)$ we have the following result:

Proposition 1. ***(IADL threshold)*** *The threshold below which IADL occurs,* $\lambda^* = \lambda^*(w_C)$*, increases with* w_C *and therefore decreases with trade openness* $1 - w_C$*.*

Proof: See appendix C.3.

Consequently, trade openness *decreases the possibility of IADL.*[14] To understand the IADL, notice that we can write Ricardian labour supply as: $n_t^R = \frac{1}{\varphi}\left(\Upsilon - \frac{\sigma}{w_C}\right) c_t^R$, which implies that hours fall in consumption for Ricardian households provided $\Upsilon < \frac{\sigma}{w_C}$. When asset market participation is sufficiently low, the profit channel dominates the wage effect, and increases in the real interest rate $r_t - \pi_{t+1} = w_C\,(r_t - \pi_{H,t+1})$ can have an expansionary effect on output y_t. For example, in the closed economy ($w_C = 1$) it follows from (3.5) that $\Xi^{w_C=1} = \frac{\Upsilon(1+1/\zeta)}{\varphi+\sigma(1+1/\zeta)} < 0$ under IADL. From (3.2) and (3.3), a rise in the real interest rate increases output by reducing Ricardian consumption c_t^R, exerting upward pressure on inflation from the NKPC. This contrasts with the standard aggregate demand logic (SADL) where both output and consumption respond negatively to real interest rate rises. In open economies, it follows from (3.4) and (3.5) that $\Xi > 0$ when $\Upsilon > 0$, so that c_t^R always increases in y_t under SADL. However, under IADL, c_t^R can either increase or decrease in y_t depending on the degree of LAMP.

The parameter Υ is a function of λ and the other model parameters w_C, φ, σ, and ω, but is independent of the monetary policy rule. For this, we initially assume the policymaker follows a simple inertial rule of the form:

$$r_t = \rho_r r_{t-1} + \theta_\pi \pi_{t+1}, \tag{3.7}$$

where $\rho_r \geq 0$ is the degree of interest rate inertia and $\theta_\pi \geq 0$ is the inflation response

[14] This result is consistent with the findings of Boerma (2014). Buffie and Zanna (2018) find trade openness can reduce the threshold value λ^* close to zero in an imperfect capital mobility model with multiple sticky-price and flexible-price sectors.

coefficient. Note that the central bank adopts a super-inertial policy if $\rho_r \geq 1$, and the integral rule with $\rho_r = 1$ yields a price-level (Wicksellian) rule. The interest-rate rule (3.7) can be expressed as:

$$r_t = \rho_r r_{t-1} - \frac{\sigma(1-w_C)\theta_\pi}{w_C} c_t^R + \frac{\sigma(1-w_C)\theta_\pi}{w_C} c_{t+1}^R + \theta_\pi \pi_{H,t+1}. \tag{3.8}$$

Equations (3.1), (3.2) and (3.8) imply the minimal state-space representation of the model:

$$\mathbf{z}_{t+1} = \mathbf{A}\mathbf{z}_t, \quad \mathbf{z}_t = \begin{bmatrix} c_t^R & \pi_{H,t} & r_{t-1} \end{bmatrix}'. \tag{3.9}$$

where the coefficient matrix $\mathbf{A}$ is given in appendix C.1.

3.1 Determinacy Analysis

We start by examining the stability properties of the model for the policy rule (3.7).

Proposition 2.

(a) ***(Role of interest-rate inertia)*** *For the standard SADL case $\lambda > \lambda^*$, interest rate inertia increases the policy space for θ_π for which there is determinacy. An equilibrium exists for all $\lambda \in (\lambda^*, 1]$ with an appropriate choice of θ_π. Under IADL, there exists some value of $\lambda \in [0, \lambda^*)$ for which a unique stable equilibrium exists. Interest rate inertia in this case reduces the policy space for θ_π, and for some values $\lambda \in [0, \lambda^*)$ if*

$$-\frac{2\sigma(1+\beta)}{\Psi w_C} < \Upsilon < -\frac{2\sigma(1+\beta)(1-w_C)}{\Psi w_C}$$

then a unique stable equilibrium does not exist for $\theta_\pi > 0$.

(b) ***(Role of trade openness)*** *For the standard SADL case $\lambda > \lambda^*$, trade openness $1 - w_C$ decreases the policy space for θ_π for which there is determinacy. Under IADL, the determinate policy space for θ_π increases with $1 - w_C$ for some values $\lambda \in [0, \lambda^*)$ if*

$$-\frac{2\sigma(1+\beta)(1-w_C)}{\Psi w_C} < \Upsilon < 0.$$

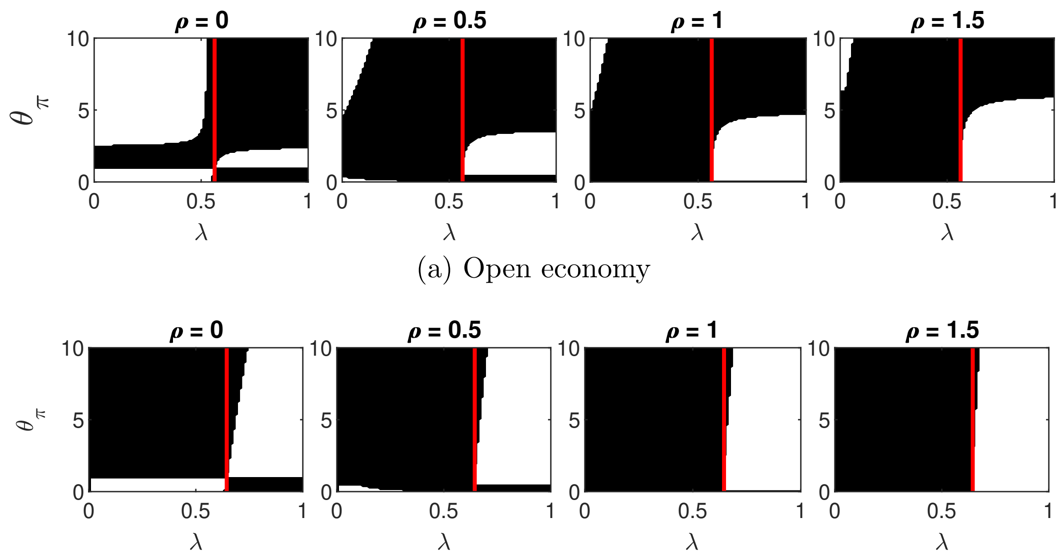
Proof: See appendix C.4.

The results given in propositions 2 follow from the necessary and sufficient conditions for equilibrium determinacy outlined in the online appendix. In the absence of interest rate inertia ($\rho_r = 0$), the Taylor principle ($\theta_\pi > 1$), which implies an 'active' policy feedback to future inflation, is a necessary condition for determinacy in the SADL case ($\Upsilon > 0$). In contrast, for the IADL case ($\Upsilon < 0$) a 'passive' policy stance ($\theta_\pi < 1$), or the *inverted Taylor principle*, is consistent with determinacy for closed economies. The determinacy conditions indicate that increasing interest rate inertia *increases* the range of determinacy under SADL, while it *reduces* the determinate policy space under IADL. Trade openness has contrasting effects. By reducing an upper bound on the inflation response coefficient, denoted Γ_1, the determinacy region shrinks in open SADL economies. However, the region of determinacy can actually increase under IADL, as the economy becomes more open. For sufficiently low values of λ, determinacy arises under the Taylor principle provided the inflation response coefficient is set sufficiently high $\theta_\pi > \max\left\{\frac{1}{1-w_C}, \Gamma_1\right\}$.

The above results are illustrated in Figure 2 for a standard quarterly parameterization. We set the discount factor $\beta = 0.99$, the CRRA coefficient $\sigma = 2$, $\zeta = 7$, implying a markup of 16 percent, and the real marginal cost elasticity of inflation $\Psi = 0.086$, consistent with an average price duration of one year. The open economy parameters are set with home bias $w_C = 0.6$ and an elasticity of substitution $\mu_C = 0.62$ in line with the estimates of Boehm et al. (2019). By inspection, while policy inertia has a stabilizing effect on the SADL economy, trade openness has a destabilizing effect. Under IADL, determinacy can also arise under the Taylor principle, as openness exerts a stabilizing effect, whereas policy inertia now destabilizes the IADL economy.

Under super-inertial policy ($\rho_r \geq 1$), determinacy is easily achieved in the SADL case. For instance, consider the closed-economy version of the model ($w_C = 1$) with price-level targeting ($\rho_r = 1$). The necessary and sufficient condition for determinacy is given by $0 < \theta_\pi < 2 + \frac{4\sigma(1+\beta)}{\Psi\Upsilon}$, where the upper bound is non-binding within the empirically-relevant interval $\theta_\pi \in [0, 10]$, except for λ very close to the threshold λ^*.[15] In contrast, determinacy is only possible under IADL if $\left(1 - \left[1 + \frac{2\sigma(1+\beta)}{\Psi}\frac{1+\varphi}{\sigma+\varphi}\right]^{-1}\right)\lambda^* < \lambda < \lambda^*$. For the baseline parameter values, there exists a very small interval of λ for which determinacy is possible ($0.986\lambda^* < \lambda < \lambda^*$). As highlighted in Figure 2, the determinacy region is barely visible under a price-level rule (and super-inertial policy in general) for both closed and open IADL economies. This is in stark contrast to the case of no rule-of-thumb consumers

[15] With our baseline parameter values, the upper bound on θ_π lies in the interval [2, 10] for $\lambda \in [0.644, 0.684]$.



(a) Open economy

(b) Closed economy

Figure 2: Determinacy regions (white areas) for the baseline LAMP model. Parameter values are $\Psi = 0.086$, $\varphi = \sigma = 2$, $\beta = 0.99$, $\zeta = 7$, and $\mu_C = 0.62$, $w_C = 0.6$ for the open economy (top panel) and $w_C = 1$ for the closed (bottom panel). The red vertical line gives λ^* below which IADL holds.

($\lambda = 1$), where determinacy is easily induced.[16]

For some intuition, first consider a sunspot-induced increase in inflationary expectations in a closed economy. For the SADL case, the Taylor principle induces a rise in the real interest rate, resulting in a fall in consumption and output. This exerts downward pressure on real marginal cost, which lowers inflation from the NKPC, contradicting the initial inflationary expectations. Similar to Bullard and Mitra (2007), interest-rate inertia helps to enlarge the determinacy region, as the long-run nominal interest-rate is $1/1-\rho_r$ times more responsive to permanent changes in inflation compared to the non-inertial case. Under super-inertial rules, any increase in inflation results in a rise in both the nominal and real interest rate. For any $\theta_\pi > 0$, the Taylor principle is always satisfied and indeterminacy is not possible.

In IADL economies, Ricardian consumption falls but output rises in response to a higher real interest rate. Consequently, real marginal cost increases and the initial inflationary belief becomes self-fulfilling under the Taylor principle. In this case, a passive policy re-

[16]For example, there exists a unique stable equilibrium in the closed economy after setting $\lambda = 1$ iff $0 < \theta_\pi < 2\left[1 + \frac{2\sigma(1+\beta)}{\Psi(\sigma+\varphi)}\right]$.

sponse by letting the real interest fall in response to higher expected inflation, leads to lower demand and deflation from the NKPC, contradicting the initial inflationary expectations. However, interest rate inertia reduces the determinacy region under the inverted Taylor principle, and determinacy becomes nearly impossible under super-inertial rules.

In open economies, the next-period consumer-price inflation rate depends on both the rate of future domestic price inflation and changes in the terms of trade:

$$\pi_{t+1} = \pi_{H,t+1} + (1 - w_C)(s_{t+1} - s_t) = \pi_{H,t+1} + \sigma \left(\frac{1 - w_C}{w_C} \right) \left(c_{t+1}^R - c_t^R \right).$$

For the SADL case, a real interest rate rise results in an expected deterioration in the terms of trade $s_{t+1} - s_t > 0$. Consequently, indeterminacy can arise under the Taylor principle provided the upward pressure on consumer-price inflation, generated by the adjustments in the terms of trade, is sufficiently strong to offset the reduction in domestic-price inflation generated from lower domestic demand. As the degree of trade openness $1 - w_C$ increases, the economy becomes more prone to indeterminacy. However, in stark contrast to closed economies, determinacy can be consistent with the Taylor principle under IADL. While rises in the real interest rate now result in an increase in domestic-price inflation, the upward pressure exerted on consumer-price inflation can be more than offset via a reduction in Ricardian consumption ($c_{t+1}^R - c_t^R < 0$) arising from the adjustment in the terms of trade.

Below we examine the robustness of these findings using several variants of the policy rule (3.8) commonly found in the literature.

3.2 Domestic-Price Inflation Targeting

We now consider the determinacy implications of rule-of-thumb consumers under a domestic price inflation rule with policy inertia:

$$r_t = \rho_r r_{t-1} + \theta_\pi \pi_{H,t+1}, \tag{3.10}$$

where setting $\rho_r = 1$ yields a domestic-price-level rule.

Proposition 3. ***(Domestic-price inflation)*** *For the standard SADL case $\lambda > \lambda^*$, interest rate inertia increases the policy space for θ_π for which there is determinacy. Under IADL, interest rate inertia decreases the determinate policy space for θ_π. The effect of* *trade openness is ambiguous. However, for a standard range of parameter values, trade openness enlarges the determinate policy space under SADL and reduces it under IADL.*

Proof: See appendix C.5.

Under a domestic-price inflation rule, the role of trade openness can be reversed. For the SADL case, the upper bound on the inflation response coefficient is now given by $\theta_\pi < (1+\rho_r)\left[1+\frac{2\sigma(1+\beta)}{\Psi w_C \Upsilon}\right] \equiv \Gamma_1^{PPI}$, which can either increase or decrease with trade openness $1-w_C$ depending on the value of $\lambda > \lambda^*$.[17] For the IADL case, the large determinacy region that arises under the Taylor principle in open economies is no longer available if domestic-price inflation is targeted.[18]

Interest-rate inertia has similar implications for determinacy regardless of the choice of inflation target. For example, consider a domestic price-level rule by setting $\rho_r = 1$ in (3.10). The necessary and sufficient condition for equilibrium determinacy is given by $0 < \theta_\pi < 2 + \frac{4\sigma(1+\beta)}{\Psi w_C \Upsilon}$. Therefore, determinacy is impossible in IADL economies provided $-2\sigma(1+\beta)/\Psi w_C < \Upsilon < 0$, which using the baseline parameter values suggests $\lambda < 0.63$ for a closed economy and $\lambda < 0.55$ with $w_C = 0.6$. In both cases, this threshold is a value approximately 0.01 below λ^*, emphasizing the narrowness of the region for which determinacy is possible.[19]

3.3 Output Stabilization

We now consider the determinacy implications of a policy response to contemporaneous output (or the output gap). Since y_t is linear in c_t^R, the Taylor rule can be expressed as:

$$r_t = \rho_r r_{t-1} + \theta_\pi \pi_{t+1} + \theta_y \Xi c_t^R, \tag{3.11}$$

where $\theta_y \geq 0$ is the output response coefficient and Ξ is given by (3.5).

Proposition 4. ***(Output targeting)*** *For the standard SADL case $\lambda > \lambda^*$, a policy response to output $\theta_y > 0$ increases the policy space of θ_π for which there is determinacy. Under IADL, there exists some values of $\lambda \in [0, \lambda^*)$ for which the Taylor principle is re-*

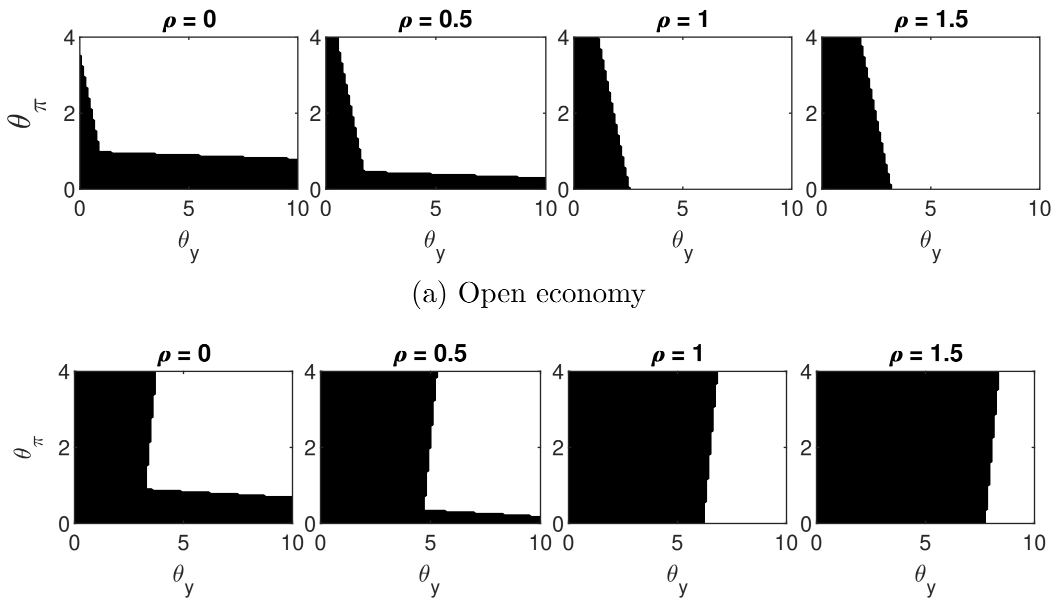
[17] Close to the IADL threshold, λ^*, the upper-bound Γ_1^{PPI} is decreasing with $1-w_C$ and in the case of no rule-of-thumb consumers ($\lambda = 1$), it is increasing with $1-w_C$ provided $1-\sigma\mu_C > 0$.

[18] Plots of the determinacy regions are shown in Figure 11 in appendix C.5.

[19] As shown in appendix C.7, similar conclusions are obtained under a contemporaneous-looking interest-rate rule: $r_t = \rho_r r_{t-1} + \theta_\pi \pi_t$.

stored. However, in this case, the equilibrium is indeterminate regardless of the value of θ_π if $\theta_y < \overline{\theta}_y$, where $\overline{\theta}_y$ is increasing with interest rate inertia and decreasing with trade openness.

Proof: See appendix C.6.



(b) Closed economy

Figure 3: Determinacy regions (white areas) under IADL for the baseline LAMP model. Parameter values are $\lambda = 0.5$, $\Psi = 0.086$, $\varphi = \sigma = 2$, $\beta = 0.99$, $\zeta = 7$, and $\mu_C = 0.62$, $w_C = 0.6$ for the open economy (top panel) and $w_C = 1$ for the closed economy (bottom panel).

Under SADL, both closed and open economies are less prone to indeterminacy with a policy response to output. Since $\Xi > 0$ with $\Upsilon > 0$, it follows that the slope $\frac{(1-\beta)\Xi}{\Psi\Upsilon}$ of the long-run NKPC is positive and the generalized (or long-run) version of the Taylor principle is given by:

$$\theta_\pi + \frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y > 1 - \rho_r. \tag{3.12}$$

Increasing ρ_r results in a parallel inward shift of the long-run Taylor principle on the plane (y_t, π_t), and the upper bound Γ_1^y on the inflation response coefficient is increasing in θ_y:

$$\Gamma_1^y \equiv (1 + \rho_r)\left[1 + \frac{2(1+\beta)\sigma w_C}{\Psi w_C \Upsilon + 2\sigma(1+\beta)(1-w_C)}\right] + \frac{w_C(1+\beta)\Xi}{\Psi w_C \Upsilon + 2\sigma(1+\beta)(1-w_C)}\theta_y.$$

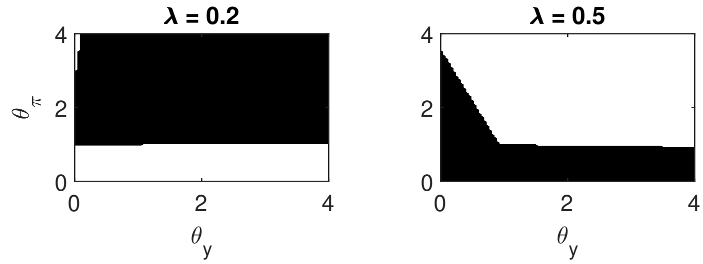


Figure 4: Determinacy regions (white areas) under IADL for the baseline LAMP model. Parameter values are $\lambda = 0.2, 0.5$, $\rho = 0$, $\Psi = 0.086$, $\varphi = \sigma = 2$, $\beta = 0.99$, $\zeta = 7$, $\mu_C = 0.62$, and $w_C = 0.6$.

With a policy response to output, the IADL breaks down in closed economies. Since $\Xi < 0$ with $\Upsilon < 0$ (after setting $w_C = 1$), determinacy can only arise under the inverted Taylor principle when $\theta_y = 0$. Instead, determinacy requires the central bank to follow the generalized Taylor principle (3.12) and place a sufficiently large weight on output: $\theta_y > \left(\frac{\theta_\pi - 1 - \rho_r}{1+\beta}\right) \frac{\Psi\Upsilon}{\Xi} - \frac{2\sigma(1+\rho_r)}{\Xi}$. However, as illustrated in Figure 3(b), determinacy requires θ_y to be large suggesting that indeterminacy is likely to arise in a closed economy when $\Upsilon < 0$ for empirically realistic output responses $\theta_y \in [0, 2]$. Moreover, since the lower bound on θ_y is increasing in ρ_r, policy inertia further undermines the ability of a policy response to output to help restore the Taylor principle when $\lambda < \lambda^*$.

In open economies Ξ can be positive or negative under $\Upsilon < 0$. This switch is clearly shown in Figure 4 by setting $\lambda = 0.2, 0.5$, since $\Xi > 0$ for any $\lambda < 0.3$ under the baseline parameter values. Thus, for low levels of λ, the IADL is maintained and determinacy arises under the inverted Taylor principle. Figure 3(a) highlights the role of trade openness and policy inertia under $\Upsilon < 0$ when $\Xi < 0$. By inspection, openness not only improves the determinacy properties of the IADL economy by lowering λ^*, but for the case $\lambda < \lambda^*$, the determinate policy space is also much larger in the open economy.

3.4 Exchange Rate Stabilization

We can show the results are also robust to a modified interest-rate rule that incorporates a policy response to either the real exchange rate q_t or changes in the nominal exchange rate $\Pi^{\mathcal{E}}_{t-1,t}$. Because the UIP condition holds, the exchange rate depreciation is equal to the lagged interest rate. Choosing the weight on this term is therefore equivalent to choosing ρ_r, leaving the results unchanged. For the real exchange rate, it is straightforward to show that under complete asset markets $q_t = \sigma\left(c_t^R - c_t^{R,*}\right)$. It follows directly that the determinacy conditions are equivalent to those with a policy response to output.

3.5 Incomplete Asset Markets, Trend Inflation, Capital and Dominant Currency Pricing

We now explore the robustness of our results for the inertial feedback rule (3.7) by introducing incomplete asset markets, positive trend inflation, and capital into the baseline model. The key differences are as follows. Under incomplete asset markets, the risk-sharing condition (4.2) is replaced by:

$$\mathbb{E}_t\left[\frac{\Lambda^R_{t,t+1}}{\Pi_{t,t+1}}\right] R_t = R^*_t \phi_B\left(\frac{\mathcal{E}_t B^*_{F,t}}{P_{H,t}Y_t}\right)\mathbb{E}_t\left[\frac{\Lambda^R_{t,t+1}}{\Pi_{t,t+1}}\Pi^{\mathcal{E}}_{t,t+1}\right], \tag{3.13}$$

where $\phi_B > 0$ controls the risk premium on foreign bonds. Wholesale firms use capital and labour to produce output under a constant-returns-to-scale technology. We assume that the law of motion for the capital stock is given by:

$$K_t = (1-\delta_K)K_{t-1} + \left[1 - \mathcal{S}(X_t)\right] I_t, \tag{3.14}$$

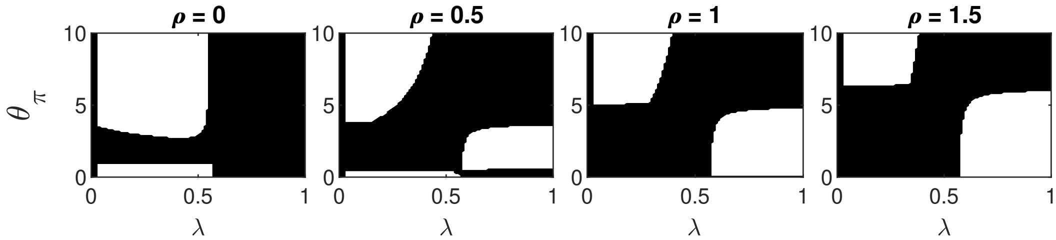
where $0 < \delta_k < 1$ and $\mathcal{S}(X_t)$ represents an adjustment cost to investment. In the steady state it is assumed that the trend inflation rate is positive, $\Pi \geq 1$. Since analytical results are not possible, we run numerical computations of local stability and determinacy.[20]

We find that the results from the determinacy analysis are not dependent on the international risk-sharing assumption and this is robust across a large range of values of ϕ_B. Figure 5 shows the determinacy regions in the model with both incomplete asset markets and an annualized trend inflation rate of 4 percent. Under IADL, the determinacy region for open economies is found to be increasing in Π, where trend inflation expands significantly the determinacy region under the Taylor principle for all values of ρ_r. This is in stark contrast to the closed IADL economy, where trend inflation has little effect on determinacy.[21]

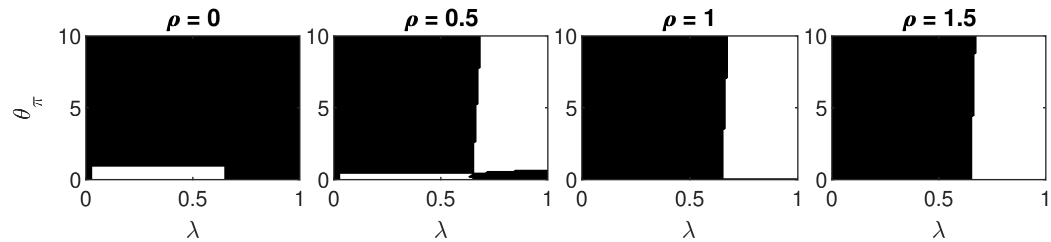
Similar conclusions are also obtained with the inclusion of capital and investment spending.

[20] Appendix B of the online appendix summarizes the complete set of equilibrium conditions for this model version. Numerical simulations use the parameter values summarized in Table 2 of appendix B.7. Additional results are provided in appendix C.8. Dynare was used to compute the results.

[21] For the standard SADL case, determinacy is not possible under trend inflation in the absence of policy inertia for both closed and open economies. However, as the value of ρ_r increases, the determinacy implications of trend inflation become minimal.



(a) Open economy



(b) Closed economy

Figure 5: Determinacy regions (white areas) for the LAMP model with incomplete asset markets and 4% trend inflation. Parameterization is given in Table 2 of appendix B.7.

For the IADL case, determinacy requires the inverted Taylor principle in the closed economy, which shrinks as ρ_r increases and completely disappears under super-inertial rules ($\rho_r \geq 1$). This is in stark contrast to the case of no rule-of-thumb consumers, where it is well known that by increasing the degree of interest rate inertia (see, e.g., Duffy and Xiao, 2011) or adopting a Wicksellian rule (see, e.g., McKnight, 2018) leads to significant determinacy gains in New Keynesian (NK) models with capital. Similar to the baseline (labour-only) model, the Taylor principle can achieve determinacy in open IADL economies provided λ is sufficiently small and θ_π is sufficiently greater than 1.

As a final sensitivity analysis we explore the determinacy implications when exports of the small open economy are priced in a dominant or local currency. In this model version, there are two retail sectors, domestic and export, where the export retail sector sets prices in the foreign currency. Details of this model version are given in the online appendix B.6.

The numerical results, which we present in appendix C.9, suggest that the role of λ and ρ_r remain unchanged under dominant currency pricing. Similar to the producer-currency-pricing baseline, these findings are also robust to the inclusion of incomplete asset markets and/or capital and investment spending.

3.6 Indeterminacy at the Zero Lower Bound

We now suppose that the interest rate is subject to a zero lower bound (ZLB) such that:

$$r_t + \bar{r} = \max\left\{0, \bar{r} + \rho_r r_{t-1} + \theta_\pi \pi_{t+1}\right\}. \tag{3.15}$$

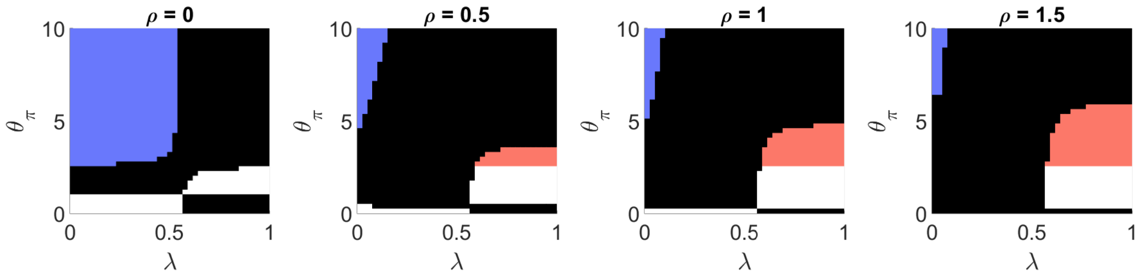
The presence of a ZLB can alter the determinacy properties of the model and introduces the possibility of both dynamic and steady-state indeterminacy.[22] Consider the following intuition for a sunspot shock induced by the ZLB. The expectation that the ZLB will bind in the future is equivalent to the expectation that for some period the nominal interest rate will be elevated above the level otherwise set by the policy rule. The higher future interest rate will have a deflationary effect and induce a cut in the interest rate today. If either the deflation is severe enough or the response of current monetary policy strong enough, then the interest rate can reach zero, and the ZLB episode would be self-fulfilling.

In order to test for equilibrium determinacy at time t, we start by choosing a future horizon $t+T$ at which point the agents believe the economy will be away from the ZLB. We then use the tests discussed in Holden (2022) to check the necessary and sufficient conditions for equilibrium determinacy for different horizons, T. If a sunspot-induced ZLB equilibrium is possible at t even if agents expect to be away from the ZLB in the following period, then the economy always suffers from indeterminacy irrespective of beliefs about the future. We might be able to rule out sunspot equilibria at t if agents believe the economy will be away from the ZLB by $t+T$, but this is not sufficient proof that there can never be sunspot equilibria. However, in principle, we could choose a value for T large enough that the risk of the ZLB binding at this future point should not plausibly affect current inflation.[23]

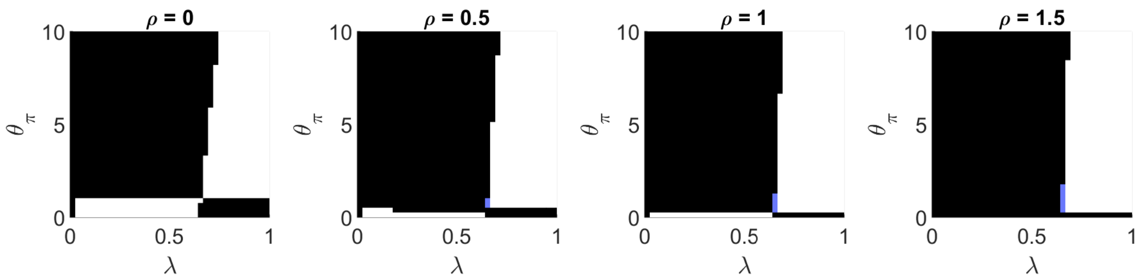
While a full check of all the necessary and sufficient conditions is too computationally expensive for large values of T, we can check some sufficient conditions with a horizon $T = 200$, which is equivalent to agents expecting to have escaped the ZLB within 50 years. This exercise reveals that when a determinate policy rule is available under IADL,

[22] It is easily verified that two deterministic steady states exist in the standard NK model with a ZLB; one steady state exists when the nominal interest rate is at zero and inflation is below target, and a second steady state arises under a positive interest rate and inflation on target. See Benhabib and Uribe (2002) and Fernández-Villaverde et al. (2015) for a detailed analysis of dynamic indeterminacy under a ZLB.

[23] A detailed discussion of the tests is provided in the online appendix. For further reading, see Holden (2022) who outlines the necessary and sufficient conditions for determinacy in an otherwise linear model with a ZLB.



(a) Open economy



(b) Closed economy

Figure 6: Uniqueness results for the baseline LAMP model with a ZLB. The black areas represent indeterminacy in the linear model, the white areas indicate there is always a unique equilibrium conditional on agents expecting to be away from the ZLB in 20 quarters. Uniqueness can only be guaranteed in the red areas when the economy escapes the ZLB in the following period. In the blue areas, self-fulfilling ZLB episodes are always possible. Parameterization is the same as in Figure 2.

uniqueness is always guaranteed except for high values of $\theta_\pi > \max\left\{\frac{1}{1-w_C}, \Gamma_1\right\}$. However, we cannot rule out multiplicity under SADL except in the absence of interest rate inertia.[24] Restricting our analysis to a shorter horizon $T = 20$, implying agents expect the economy to have escaped the ZLB in 5 years, allows us to check the full set of necessary and sufficient conditions by employing the recursive test proposed in Tsatsomeros and Li (2000).[25]

Figure 6 shows the results of these tests. For open IADL economies, the blue region arising from a sufficiently large inflation response θ_π suffers from the risk of equilibrium multiplicity. Here, the nominal interest rate responds negatively to a positive contemporaneous monetary policy shock and self-fulfilling ZLB episodes are always possible due to the ag-

[24]It turns out that uniqueness under SADL with no policy inertia is a knife-edge result that does not hold in the medium-scale model.

[25]We rely on the implementation of these tests in the dynareOBC toolkit (see `https://github.com/tholden/dynareOBC`) as described in Holden (2022).

gressiveness of the policy rule. The red area shows the region in the parameter space for which indeterminacy arises from the ZLB in open SADL economies. Although not quite as unstable as the blue region of the IADL economy, multiple equilibria arises unless the economy is expected to be away from the ZLB in the following period. Consequently, the determinate policy space shrinks in open SADL economies with policy inertia, where multiple equilibria can occur as a result of future news of the ZLB binding.

This might seem to contradict that existing literature which finds that price-level targeting and other make-up strategies can prevent sunspot equilibria, but notice that the lag of the interest rate in (3.15) will be zero when at the ZLB. Any price-level information stored is therefore lost when the ZLB is binding. We can retain this either with a defined price-level target or if we include a shadow interest rate r_t^* in the interest rate rule:

$$\begin{aligned} r_t + \bar{r} &= \max\{0, r_t^* + \bar{r}\} \\ r_t^* &= \rho_r r_{t-1}^* + \theta_\pi \pi_{t+1}. \end{aligned} \tag{3.16}$$

Under this policy rule, determinacy is restored to the red regions highlighted in Figure 6. This highlights that policy inertia is not enough to guard against ZLB risk.

We can look further at how policy inertia and trade openness affect the determinacy properties of the model under a ZLB using other indicative statistics. For the interest-rate rule (3.15), except for small values of ρ_r, we find that higher policy inertia worsens the determinacy properties of both the closed and open economy versions of the model. However, by including the lagged shadow rate (3.16), policy inertia tends to improve the determinacy conditions.[26] Consider the following intuition. As already discussed, the presence of self-fulfilling ZLB episodes depends on the current impact of future monetary policy news shocks. Policy inertia can have two competing effects in this regard. On one hand, policy inertia increases the persistence of monetary policy shocks, implying that the ZLB binding is more contractionary in the presence of inertia, increasing the risk of sunspots. On the other hand, under inertia, a change in the interest rate will move long-term interest rates, thus having a larger impact on current inflation through the expectation channel. In the case of a shadow rate rule, the higher inflation expectations under policy inertia offsets the contractionary effect of news of future ZLB episodes.

[26] See appendix C.10.1 for details.

4 Optimal Monetary Policy

This section considers optimal policy using a (slightly) restricted form of the model for reasons of analytical tractability. The optimal policy problem is defined in Section 4.3. We confine ourselves to the case where the policymaker can commit (as for the Taylor-type rules examined above). As is standard in the literature, in order to derive analytical results, we define an approximate linear-quadratic optimal policy problem. Section 4.4 sets out the commitment solution where the policymaker is concerned about interest rate variance.

We follow Galí and Monacelli (2005), among others, and restrict the welfare analysis to the special case where $\sigma = \mu_C = \mu_C^* = 1$.

Assumption 1. ***(Restricted parametrization)*** *We hereafter assume: (i) log utility in consumption ($\sigma = 1$); (ii) unit elasticity of substitution between home and foreign goods ($\mu_C = 1$); (iii) unit elasticity of substitution between goods produced in the RoW ($\mu_C^* = 1$); (iv) no fixed costs ($F = 0$) so without subsidies, the steady state is not equitable.*

In our model there are three market distortions. In addition to market power arising from monopolistic competition and relative price dispersion arising from nominal price stickiness, the terms of trade can be influenced to the benefit of domestic consumers. Moreover, with LAMP, there is an additional behavioral distortion which creates inequality across household types.[27]

4.1 The Symmetric Equilibrium with the Restricted Parametrization

With the restricted parametrization of Assumption 1 the CPI index, risk sharing and output equilibrium conditions respectively become

$$P_t = P_{H,t}^{\mathrm{w}_C} P_{F,t}^{1-\mathrm{w}_C} \tag{4.1}$$
$$C_t^R = C_t^{R\,*} Q_t \tag{4.2}$$
$$Y_t = C_t \left(\frac{P_{H,t}}{P_t}\right)^{-\mu_C} \tag{4.3}$$

where we recall the real exchange rate $Q_t \equiv \frac{\mathcal{E}P_t^*}{P_t} = \frac{P_{F,t}}{P_t}$ (assuming producer currency pricing) and the terms of trade $S_t = \frac{P_{F,t}}{P_{H,t}}$. Hence from (4.1), $Q_t = S_t^{\mathrm{w}_C}$ so (4.2) and (4.3)

[27] This distortion arises from three sources preventing constrained households from (i) owning domestic shares, (ii) owning foreign shares, and (iii) trading in international state-contingent securities.

combine to give

$$C_t^{w_C}(C_t^R)^{1-w_C} = (C_t^{R*})^{1-w_C} Y_t^{w_C} \tag{4.4}$$

4.2 The Social Planner's Equitable Allocation Problem

The social planner's problem for the SOE with LAMP is to choose C_t^i and N_t^i for $i = C, R$ to maximize aggregate utility $\lambda U(C_t^R, N_t^R) + (1-\lambda)U(C_t^C, N_t^C)$ subject to the resource constraint. We seek an equitable allocation that removes the LAMP distortion to give constrained consumers the benefit of risk-sharing. Then the optimal equitable allocation with $C_t = C_t^R = C_t^C$ and $N_t = N_t^R = N_t^C$ follows from optimizing the same aggregate utility function but subject to the risk-sharing and output equilibrium constraint given by (4.4) and the technology constraint $Y_t = \frac{A_t N_t}{\Delta_t}$. In the optimal allocation prices are flexible so the price dispersion $\Delta_t = 1$. In terms of C_t and N_t and given C_t^{R*} the relevant constraint becomes

$$C_t = (A_t N_t)^{w_C}(C_t^{R*})^{1-w_C}. \tag{4.5}$$

The first-order condition then is given by

$$U_{N,t} + U_{C,t} C_t^{R*} A_t^{w_C} w_C N_t^{w_C - 1} = 0 \tag{4.6}$$

With our choice of preferences (2.1) (with $\sigma = 1$), $U_{N,t} = N_t^{\varphi}$ and $U_C = \frac{1}{C_t}$. Then combining (4.5) and (4.6) we arrive at

$$N_t = N = w_C^{\frac{1}{1+\varphi}} \tag{4.7}$$

Thus the socially optimal labour supply is constant. The deterministic steady state of this equilibrium is the baseline about which the first-order solution of the model and the second-order approximation of the welfare criterion are conducted, in accordance with the equilibrium determinacy analysis of section 3.

How then can the decentralized equilibrium sticky-price SOE with LAMP support this optimal equitable allocation in our baseline steady state? We seek two tax instruments, a firm subsidy τ_f that eliminates the sticky-price distortion and a household subsidy τ_h that eliminates the LAMP distortion. Both are financed out of lump-sum taxation of Ricardian consumers. These payments must then satisfy

$$W(1-\tau_f) = -\frac{U_{N^i}}{U_{C^i}}; \qquad i = R, C, \tag{4.8}$$

$$C^C = W(1+\tau_h)N^C. \tag{4.9}$$

From the sticky-price decentralized equilibrium this requires tax subsidies that satisfy:

$$\mathrm{w}_C(1-\tau_f) = 1 - \frac{1}{\zeta}, \tag{4.10}$$

$$1+\tau_h = \frac{1}{\mathrm{w}_C}. \tag{4.11}$$

Thus the next proposition directly follows.

Proposition 5. ***(Subsidies for an Optimal Equitable Allocation)*** *Given the sticky-price LAMP equilibrium, in our baseline steady state an optimal equitable flexi-price allocation is sustained following* (4.10) *and* (4.11) *which determine tax subsidies for the firm* τ_f *and household* τ_h*. These subsidies are financed by lump-sum taxes, introduced in the budget constraint for Ricardian households* (2.3)*.*

Note that the LAMP dimension, via λ, does not appear in either (4.10) and (4.11). The optimal employment subsidy paid to the firm is influenced (negatively) by the degree of trade openness, $1-\mathrm{w}_C$, in addition to its standard (positive) dependence on the inverse of the markup, $1-1/\zeta$. In contrast, the optimal wage subsidy paid to all households is positively related to the degree of trade openness. These results generalize the results of Bilbiie (2008) for the closed LAMP economy ($\mathrm{w}_C = 1$), where no household subsidy is required, and Galí and Monacelli (2005) for the open economy case without LAMP ($\lambda = 1$).

4.2.1 Discussion

A result we have established here is that the optimal equitable hours of work (4.6) depend on the degree of trade openness: the more open is the economy, the less R and C agents work. This result arises because of the same risk-sharing condition (4.4) across Ricardian consumers in the SOE and the RoW. Our interpretation is linked to the role of the open-economy dimension in risk-sharing seen clearly here: the benefit of foreign profits and international risk-sharing, originally going only to the R-types, now gets shared between both R and C agents via the redistribution that makes the allocation equitable. The more open an economy, the wider the range of risk-sharing.

4.3 The Optimal Policy Problem with Commitment

The optimal policy problem consists of minimizing the second-order approximation to social welfare loss, given the constraints embodied in the model economy, summarized by the intertemporal IS equation (NKIS) and the NKPC of Section 3. For the remainder of the optimal policy analysis, along with the restrictions in Assumption 1, we choose the steady state of the determinacy analysis of Section 3 corresponding to the optimal equitable allocation. As is standard in the literature, we rewrite these equations in terms of the output gap x_t and the natural rate of interest r_t^n. From appendix C.1 we have:

$$x_t = \mathbb{E}_t x_{t+1} - w_C \Xi \left(r_t - \mathbb{E}_t \pi_{H,t+1} - r_t^n \right), \tag{4.12}$$

$$\pi_{H,t} = \beta \mathbb{E}_t \pi_{H,t+1} + \kappa x_t + u_t, \tag{4.13}$$

where the parameters $\Xi = \Xi(\lambda, w_C)$ and $\kappa = \kappa(\lambda, w_C)$ determine the slopes of the NKIS and NKPC curves given by:

$$\Xi = \frac{\lambda}{w_C(\lambda - (1-\lambda) w_C \varphi)}, \tag{4.14}$$

$$\kappa \equiv \frac{\Psi \Upsilon}{\Xi} = \Psi \frac{(1 + w_C \varphi)\lambda + (1 - w_C)\varphi}{\lambda}, \tag{4.15}$$

$$\Upsilon = \frac{(1 + w_C \varphi)\lambda + (1 - w_C)\varphi}{w_C(\lambda - (1-\lambda) w_C \varphi)}, \tag{4.16}$$

$$\Psi \equiv \frac{(1-\xi)(1-\beta\xi)}{\xi}. \tag{4.17}$$

In (4.13) we have added a cost-push mark-up shock process u_t, which in logs is of AR(1) form, $u_t = \rho_u u_{t-1} + \epsilon_t$, where ϵ_t is i.i.d. Note also that given the restrictions of Assumption 1 equation (3.4) becomes (4.16).[28] It follows that the threshold λ^* at which IADL occurs:

$$\lambda < \lambda^* = \frac{w_C \varphi}{1 + w_C \varphi}. \tag{4.18}$$

[28] In particular, setting $\sigma = 1$ and $F = 0$ implies $\zeta = \infty$.

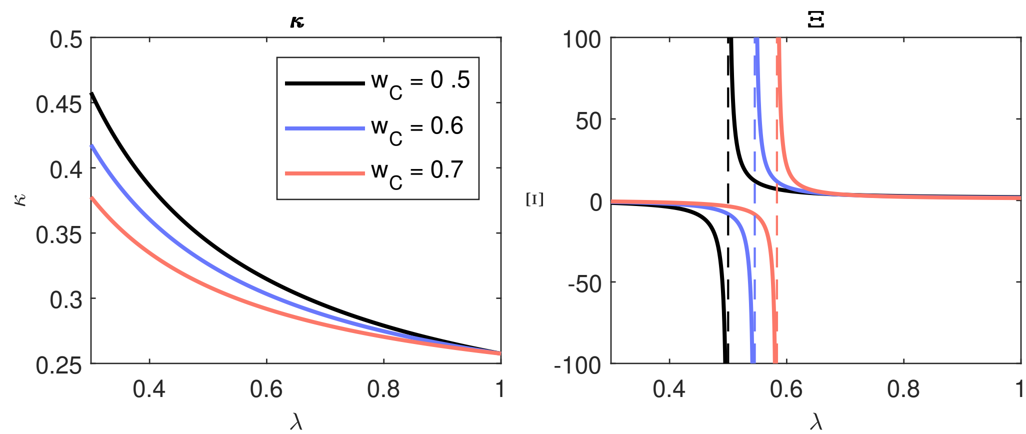


Figure 7: Role of LAMP (λ) and trade openness ($w_C = 0.5, 0.6, 0.7$) for the slopes of the NKPC and NKIS, respectively. Parameter values are $\Psi = 0.086$, $\varphi = 2$, $\beta = 0.99$, and $\sigma = \mu_C = 1$. The values of λ^* for each value of w_C are shown with the dashed lines in the right-hand chart.

Recall from Section 3 that the slopes of the NKIS and NKPC are affected both by λ and w_C, which operate via the composite parameters κ and Ξ, where $\frac{\partial \kappa}{\partial w_C}, \frac{\partial \kappa}{\partial \lambda} < 0$ and $\frac{\partial \Xi}{\partial w_C}, \frac{\partial \Xi}{\partial \lambda} < 0$. Figure 7 illustrates the role of trade openness and LAMP for the NKIS and NKPC, setting $\sigma = \mu_C = 1$, $\beta = 0.99$, $\varphi = 2$, and $\Psi = 0.086$. For any given w_C and inverse Frisch elasticity φ, when gradually increasing the degree of LAMP from nil (at $\lambda = 1$), at some point the sign of the NKIS curve becomes positive (as Ξ becomes negative). The intuition is that the more open the economy or the higher the degree of LAMP, the less domestic output depends on the domestic real interest rate. The latter is because constrained consumers spend their current income *irrespective* of the interest rate. Observe that for this restricted parameterization the slope, but now not the sign, of the NKPC is affected by λ and w_C, so that the output gap exerts greater influence on domestic inflation in the SOE with LAMP. The intuition is straightforward: the more open the economy, the more domestic inflation depends on the domestic output gap via the aggregate demand effect of increased spending on imports; and the higher the degree of LAMP, the more domestic inflation depends on the domestic output gap due to a greater share of C-type households consuming all current income, thus strengthening the link between output and inflation.

In light of Proposition 5 we now choose our social welfare criterion and by implication the welfare-relevant output gap.

Assumption 2. ***(Social welfare criterion)*** *Our social welfare criterion is a second-*

order approximation of the sum of the Ricardian and constrained households utility weighted by their mass in the region of the optimal equitable flexible-price allocation $\bar{Y}_t$ with a welfare-relevant output gap $x_t = \frac{Y_t - \bar{Y}_t}{\bar{Y}_t}$ supported by the subsidy scheme of Proposition 5.

The form of this welfare criterion is given by the following proposition:

Proposition 6. ***(Social welfare loss with a flexi-price equilibrium)*** *For the non-linear model of Section 2 and welfare-relevant output gap x_t, given Proposition 5, the micro-founded social welfare loss criterion for the LAMP SOE is approximated as:*

$$\Omega_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} (\pi_{H,t}^2 + \varpi x_t^2) - \Lambda_x x_t \right], \tag{4.19}$$

where $\varpi = \varpi(\lambda) = \frac{\Psi(1+\varphi)}{\zeta\lambda}$, $\Psi \equiv \frac{(1-\beta\xi)(1-\xi)}{\xi}$, $\Lambda_x = \frac{(1-w_C)(1-\lambda)\varphi}{\lambda}$.

Proof: See appendix D.1.

The linear term in x_t captures the fact that any marginal increase in the output gap relative to its steady state value has a positive first-order effect on social welfare, since output is below its efficient level at that steady state.

We now turn to the policy implications of our results with respect to the central bank operating under commitment, before deriving the corresponding optimal targeting rule.

4.4 Optimal Policy with Interest Rate Inertia

Nominal interest rate inertia can be introduced by penalizing its variance. The Lagrangian function for the optimization problem under commitment is now given by:

$$\begin{aligned} \mathcal{L}^C \left(\{x_t, \pi_t\}_{t=0}^{\infty}; \{\mu_t\}_{t=0}^{\infty}; \{\nu_t\}_{t=0}^{\infty} \right) \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[\tfrac{1}{2} \left(\pi_{H,t}^2 + \varpi x_t^2 + w_r r_t^2 \right) - \Lambda_x x_t \\ + \mu_t \left(\pi_{H,t} - \kappa x_t - \beta \pi_{H,t+1} \right) + \nu_t \left(x_t - x_{t+1} + \Xi w_C \left(r_t - \pi_{H,t+1} - r_t^n \right) \right) \Big] + t.i.p., \end{aligned} \tag{4.20}$$

for the welfare-relevant output gap x_t, where $\{\mu_t\}_{t=0}^{\infty}$ and $\{\nu_t\}_{t=0}^{\infty}$ are sequences of Lagrange multipliers for $t = 0, 1, 2, ...$, and the law of iterated expectations has been applied to eliminate the conditional expectation that appeared in each constraint. The penalty associated with the variance of the interest rate can be formalized in terms of an approximation to the ZLB on the nominal interest rate (see Giannoni and Woodford, 2003; Levine, McAdam and Pearlman, 2008; Woodford, 2003*a*, chap 6).[29]

Differentiating the Lagrangian function with respect to the decision variables, we obtain the respective first-order conditions:

$$\frac{\partial \mathcal{L}^C\left(\{x_t, \pi_{H,t}\}_{t=0}^{\infty}; \{\mu_t\}_{t=0}^{\infty}; \{\nu_t\}_{t=0}^{\infty}\right)}{\partial x_t} = \varpi x_t - \kappa\mu_t - \Lambda_x + \nu_t - \frac{1}{\beta}\nu_{t-1} = 0, \tag{4.21}$$

$$\frac{\partial \mathcal{L}^C\left(\{x_t, \pi_{H,t}\}_{t=0}^{\infty}; \{\mu_t\}_{t=0}^{\infty}; \{\nu_t\}_{t=0}^{\infty}\right)}{\partial \pi_{H,t}} = \pi_{H,t} + \mu_t - \mu_{t-1} - \frac{\Xi w_C}{\beta}\nu_{t-1} = 0, \tag{4.22}$$

$$\frac{\partial \mathcal{L}^C\left(\{x_t, \pi_{H,t}\}_{t=0}^{\infty}; \{\mu_t\}_{t=0}^{\infty}; \{\nu_t\}_{t=0}^{\infty}\right)}{\partial r_t} = w_r r_t + \Xi w_C \nu_t = 0, \tag{4.23}$$

which must hold for $t = 0, 1, 2, ...$, where $\mu_{-1} = 0$ and $\nu_{-1} = 0$.[30] Equations (4.21)–(4.23) plus the NKIS and NKPC equations yield an equilibrium in the multipliers μ_t and ν_t, and endogenous variables r_t, $\pi_{H,t}$, and x_t.

Proposition 7. *Define*

$$d_t = \pi_{H,t} + \frac{1}{\kappa}(1 - L)(\varpi x_t - \Lambda_x) = \pi_{H,t} + \frac{1}{\kappa}\varpi(x_t - x_{t-1}) \tag{4.24}$$

as a departure from the 'leaning against the wind' optimal condition (or 'wedge'), where $\pi_{H,t} + \frac{1}{\kappa}\varpi(x_t - x_{t-1}) = 0$ *in the case of no penalizing of the interest rate variance when* $w_r = 0$. *Then the optimal policy with commitment is given by:*

$$r_t = \left(1 + \frac{1}{\beta} + \frac{\kappa\Xi w_C}{\beta}\right) r_{t-1} - \frac{1}{\beta} r_{t-2} + \frac{\kappa\Xi w_C}{w_r} d_t. \tag{4.25}$$

Optimal policy can be implemented by the following approximate first-order dynamic inertial Taylor-type rule that responds only to the wedge:[31]

$$r_t = \left(1 + \frac{\kappa\Xi w_C}{\beta}\right) r_{t-1} + \frac{\kappa\Xi w_C}{w_r} d_t. \tag{4.26}$$

[29] We can also motivate the loss from interest rate volatility as stemming from non-modeled features other than the ZLB that provide incentive to smooth interest rates. See, e.g., Rudebusch and Svensson (1999), Woodford (2003*b*), and Givens (2012).

[30] The last equality results because the inflation first-order condition corresponding to period −1 is not an effective constraint to the monetary authority when choosing its optimal policy plan in period 0.

[31] Another approximation to the optimal rule, only available under SADL, which can be generalized to higher-order responses to the wedge, is shown in appendix D.2.

Proof: See appendix D.2

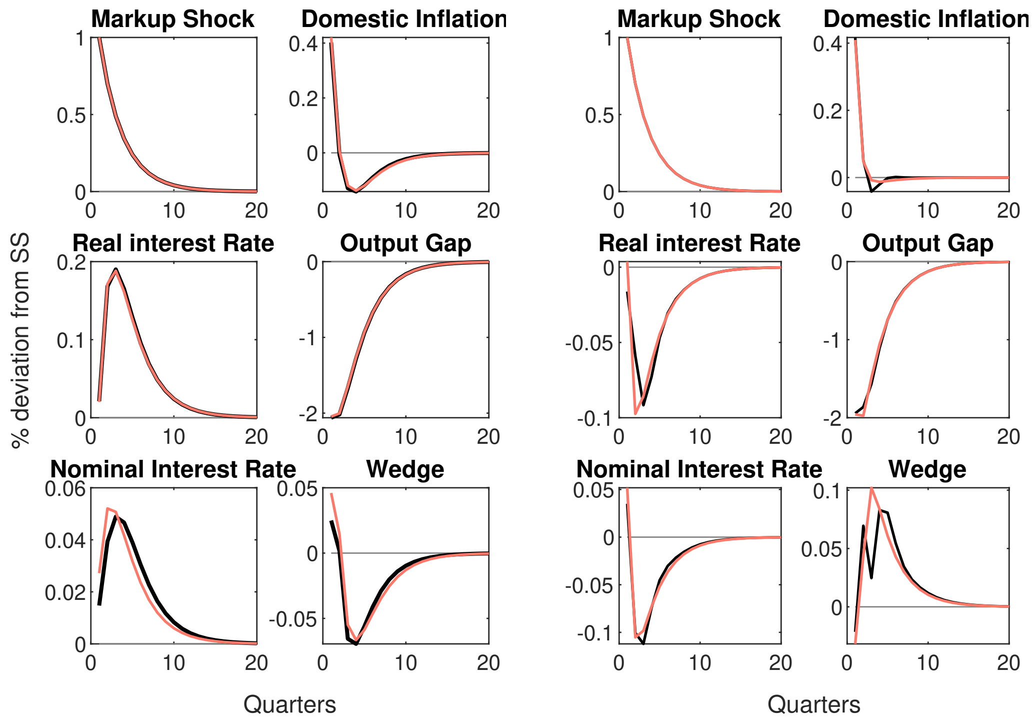
Rule (4.25) is the open-economy version of the *robustly optimal* and *timeless* implicit instrument rule derived in Giannoni and Woodford (2003) for closed economies. It is robust in the sense that it is independent of the nature of the exogenous disturbances. It is timeless in the sense that is the same for all periods $t \geq 0$, as opposed to the optimal time-variant, once-and-for-all commitment to (4.25) from $t \geq 2$ with $\mu_{-1} = \nu_{-1} = 0$ implying a different setting in periods $t = 0$ and $t = 1$. Under the restricted parameterization of the model, the optimal rule (4.25) takes the form of an inertial Taylor-type interest-rate rule (4.26) targeting domestic inflation and output gap growth.[32]

It follows that both the optimal rule (4.25) and the inertial Taylor-type rule (4.26) is super-inertial under SADL (since $\Xi > 0$), whereas the degree of inertia is significantly lower and not super-inertial under IADL. The degree of (super) inertia is independent of the weight w_r on the variance of the nominal interest rate r_t, and depends on the discount factor β, the degree of openness $1 - w_C$, and the intrinsic dynamics driving the NKPC and NKIS equations via the parameters κ and Ξ, respectively. It is the latter that determines the transmission of changes to r_t on outcomes, whilst the optimal choice of such changes only depends on w_r through the response to the wedge d_t.

How accurate is the inertial Taylor-type rule given by (4.26) as a representation of the fully optimal policy rule (4.25) with a variance penalty? Figure 8 addresses this question by comparing the optimal deviations from the equitable steady state under (4.25) and (4.26) in response to a positive mark-up shock. We use the restricted parameter values outlined in Section 4.2 setting $w_r = 1$ with the persistence of the shock equal to $\rho_u = 0.7$. Panel (a) illustrates the SADL case ($\lambda > \lambda^*$) and panel (b) depicts the IADL case ($\lambda < \lambda^*$). Notice that the optimal policy rule (4.25) can generate oscillatory behaviour under IADL, which becomes more pronounced the further the rule departs from $d_t = 0$. This is particularly true for either high levels of LAMP or high values of w_r where the interest rate is determined less by inflation and output and more by history dependence. In these cases, the Taylor-type rule (4.26) smooths out the oscillations, while still delivering very similar outcomes for output and inflation as the optimal policy rule.[33]

[32]In appendix D.3 we show how the implicit instrument rule (4.26) can be implemented in the form shown in the determinacy analysis of Section 3.

[33]This is demonstrated by Figures 20 and 21 in appendix D.4 that depict the simulations under the optimal policy rule (4.25) for variations in both the variance penalty and trade openness.

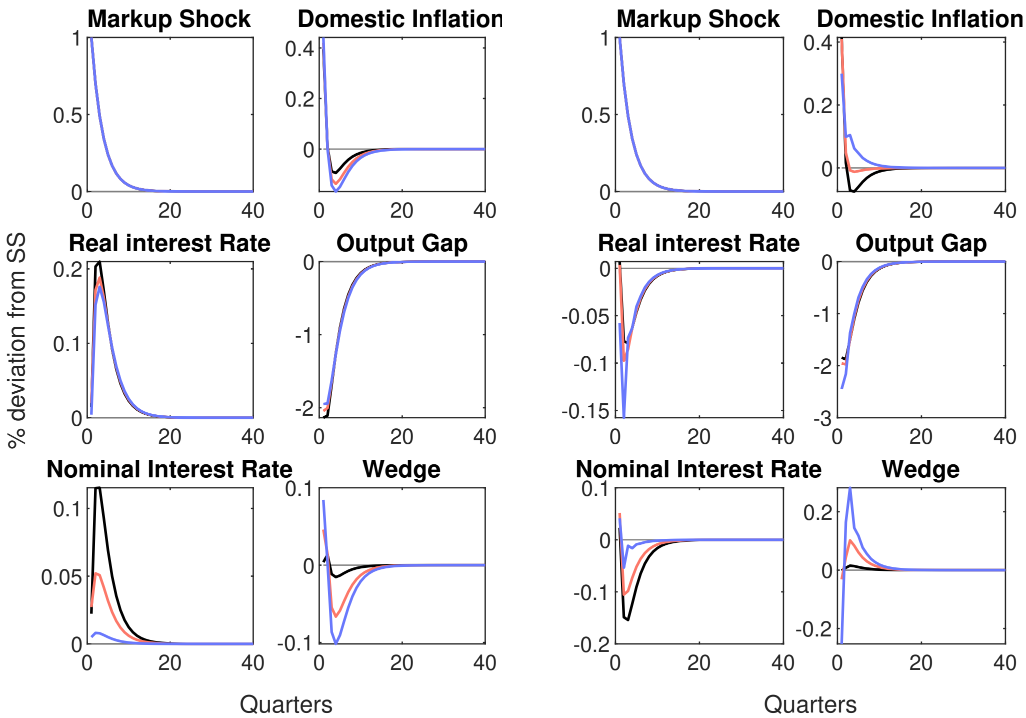


(a) SADL ($\lambda = 0.7 > \lambda^*$) (b) IADL ($\lambda = 0.5 < \lambda^*$)

Figure 8: Optimal monetary policy: optimal rule (black line) vs approximation (red line). Parameter values are $\lambda = 0.5, 0.7$, $\Psi = 0.086$, $\varphi = 2$, $\beta = 0.99$, $\sigma = \mu_C = w_r = 1$, and $w_C = 0.6$.

In the aftermath of a mark-up shock, two optimal policy trade-offs arise. In the first trade-off, determined by w_r, greater interest rate stability comes at the cost of reduced stabilization of the wedge d_t. The second is the standard trade-off, where lower inflation variability comes at the expense of greater output gap volatility, the outcome of which is determined by the standard 'leaning against the wind' policy, and the relative weight of these variables given in equation (4.24). Below, we consider how these monetary policy trade-offs are affected by the degree of trade openness and LAMP.

To evaluate the first trade off, we turn to simulations in response to a positive mark-up shock with persistence $\rho_u = 0.7$ using the restricted parameterization of Section 4.2. For the approximation rule (4.26), Figure 9 displays the effect of higher interest-rate stabilization brought about by increasing the penalty parameter w_r for the cases of $\lambda > \lambda^*$ (SADL) and $\lambda < \lambda^*$ (IADL), where λ^* is the IADL threshold given by (4.18), which equals $\lambda^* = 0.546$ with $w_C = 0.6$ and $\varphi = 2$. In both the SADL and IADL cases, an increase in the



(a) SADL ($\lambda = 0.7 > \lambda^*$) (b) IADL ($\lambda = 0.5 < \lambda^*$)

Figure 9: Optimal policy (approximation rule) and variance penalty for $w_r = 0.1$ (black line), $w_r = 1$ (red line) and $w_r = 10$ (blue line). Parameter values are $\lambda = 0.5, 0.7$, $\Psi = 0.086$, $\varphi = 2$, $\beta = 0.99$, $\sigma = \mu_C = 1$, $w_C = 0.6$.

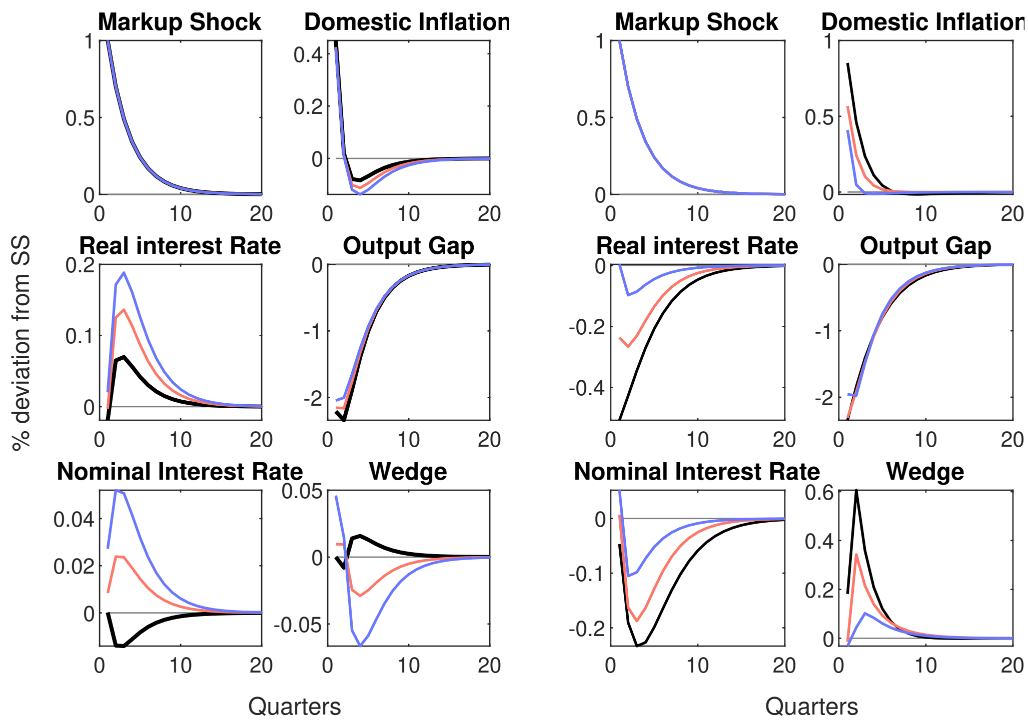
penalty dampens the monetary policy response. While domestic (and CPI) inflation is accommodated to limit the fall in output,[34] the wedge d_t moves further from its optimal value (i.e., $d_t = 0$) as the penalty parameter increases, resulting in a significant difference between the SADL and IADL cases.

Table 1 reports the optimal policy coefficients ρ_r, θ_π, θ_x of the rule (4.26) for different values of λ and w_C. Under SADL an increase in the share of hand-to-mouth behaviour calls for higher values for θ_π and θ_x and a stronger degree of super-inertia. Under IADL, super-inertial policy is not optimal, and the coefficients for θ_π and θ_x turn negative. While the parameter governing policy inertia in (4.25) or (4.26) is also increasing with the degree of LAMP in the IADL case, for values of λ below but close to λ^*, the degree of policy

[34]Since no efficiency shocks in the model are used in this section, the natural rate of output remains at its steady state ($x_t = y_t - y_t^n = y_t$). Therefore, the output gap equals output.

	$\lambda = 0.9$			$\lambda = 0.7$			$\lambda = 0.5$			$\lambda = 0.3$		
$w_C =$	1	0.8	0.6	1	0.8	0.6	1	0.8	0.6	1	0.8	0.6
$r_t = \rho_r r_{t-1} + \theta_\pi \pi_{H,t} + \theta_x (x_t - x_{t-1})$												
ρ_r	1.33	1.32	1.31	2.82	1.87	1.60	0.74	0.51	-0.65	0.93	0.88	0.77
θ_π	0.33	0.32	0.31	1.80	0.87	0.59	-0.26	-0.49	-1.63	-0.07	-0.12	-0.23
θ_x	0.05	0.05	0.05	0.37	0.17	0.11	-0.07	-0.12	-0.37	-0.03	-0.04	-0.07

Table 1: Coefficients for the inertial Taylor-type rule approximation to optimal policy. Parameterization is the same as in Figure 9. For $\lambda = 0.9, 0.7 > \lambda^*$, whereas for $\lambda = 0.5, 0.3 < \lambda^*$.



(a) SADL ($\lambda = 0.7 > \lambda^*$) (b) IADL ($\lambda = 0.5 < \lambda^*$)

Figure 10: Optimal policy (approximation rule) and openness for $w_C = 1$ (black line), $w_C = 0.8$ (red line) and $w_C = 0.6$ (blue line). Parameter values are $\lambda = 0.5, 0.7$, $\Psi = 0.086$, $\varphi = 2$, $\beta = 0.99$, and $\sigma = \mu_C = w_r = 1$.

inertia can become negative and so in this region an increase in the share of hand-to-mouth behaviour can lower the degree of history dependence.

In Figure 10 we examine the effect of trade openness on optimal policy for the SADL and

IADL cases, again using the approximation rule (4.26) setting $w_r = 1$.[35] In open economies, the exchange rate provides an additional channel for the transmission of monetary policy. For the SADL case, the exchange rate appreciation further reinforces the monetary contraction, reducing the optimal interest rate response to higher inflation. Consequently, both the optimal rule (and its approximation) exhibit a smaller response to inflation and output, and a lower degree of super-inertia, the more open the economy becomes. At the same time, trade openness weakens the link between output and the domestic real interest rate, which eases the trade-off in open economies under SADL and permits the larger monetary policy response seen in Figure 10. The opposite is true under IADL. Since the real interest rate falls in this case, the optimal policy generates a depreciation of the exchange rate, which requires a larger response to inflation and output, as the degree of openness increases. At the same time, increasing trade openness gives real interest rate adjustments a larger impact on output in the IADL case. This magnifies the impact of policy adjustments and leads to the smaller policy response in open economies shown in Figure 10.

Finally, regarding the trade-off between inflation and output stabilization, we find that this trade-off is unaffected by the IADL threshold. In both SADL and IADL economies, a higher degree of trade openness requires greater stabilization of domestic inflation relative to the output gap, whereas a higher degree of LAMP requires the opposite. The intuition is as follows. The more open is the economy, the less the output gap depends on domestic inflation, which eases the trade-off allowing the central bank to focus more on inflation. In contrast, the higher the share of hand-to-mouth behaviour, the larger the movements in demand which worsens the trade-off with the opposite effect. This reinforces the earlier work of Bilbiie (2008) and Iyer (2016) who both consider optimal monetary policy but in the absence of inertia.[36]

[35] For the restricted parameterization of Section 4.2, the IADL threshold λ^* given by (4.18) equals $\lambda^*(w_C) = 0.546, 0.615, 0.667$ under $w_C = 0.6, 0.8, 1.0$. We choose $\lambda = 0.7$ (SADL) and $\lambda = 0.5$ (IADL) to represent the two cases.

[36] Bilbiie (2008) finds the optimal response to inflation is decreasing in the degree of LAMP for a closed economy. Iyer (2016) shows that the output gap should be stabilised relatively more in a SOE with higher LAMP, in order to smooth the disposable income of hand-to-mouth agents.

5 Concluding Remarks

This paper examines the role of limited asset market participation, trade openness, and interest-rate inertia in the design of monetary policy. These features are empirically-relevant and are shown to have important considerations for policy.

Our main findings challenge the conventional wisdom that policymakers should engage in interest-rate smoothing in two important ways. First, it is well established that policy inertia helps increase the likelihood of determinacy in the absence of LAMP. In contrast, we have shown for IADL economies that determinacy is actually undermined if the central bank follows a policy rule with excessive interest-rate inertia. Therefore, the commonly-advocated use of super-inertial feedback rules, including price-level (Wicksellian) rules, as potential remedies for indeterminacy, are strongly ill-advised under LAMP.

Second, in the absence of hand-to-mouth households, optimal monetary policy is robust and timeless with super-inertia, the latter arising from the costs of interest rate volatility. We have shown that super-inertial policy is not optimal in IADL economies, and for empirically-plausible values of LAMP, a negative weight for the interest-rate smoothing coefficient can be optimal. It is important to stress the key role trade openness plays in our analysis. It exerts a stabilizing effect in IADL economies, reducing the possibility of self-fulfilling beliefs. Moreover, as emphasized in the optimal monetary policy analysis, the inertial coefficient of the optimal targeting rule crucially depends on the degree of trade openness.

Our paper has some limitations. As is standard in the literature, we used a restricted parameterization of a stylized SOE economy, in order to obtain analytical results for optimal monetary policy. However, it would be important to consider how our results change using an extended model that, among other things, allows for capital, incomplete asset markets, and local currency pricing. For example, using the full unrestricted SOE LAMP model set out in Section 3.5, it is possible to compute simple optimized rules, choosing coefficients on the variables of interest to minimize the welfare-theoretic social loss function. The policy choice is then between alternative simple targeting rules, the ranking of which crucially depends on the estimates of model parameters and shock processes. While this takes us beyond the scope of this paper, it should be explored in future research.

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Technical Appendix: Derivations and Proofs (for online publication)

A Equilibrium Conditions for Baseline LAMP Model

A.1 Households: Aggregate Consumption and Labour

$$C_t = \lambda C_t^C + (1-\lambda) C_t^R$$
$$N_t = \lambda N_t^C + (1-\lambda) N_t^R$$
$$\frac{(N_t^C)^{\varphi}}{(C_t^C)^{-\sigma}} = W_t$$
$$\frac{(N_t^R)^{\varphi}}{(C_t^R)^{-\sigma}} = W_t$$
$$C_t^C = W_t N_t^C$$
$$1 = \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}^R}{\Pi_{t,t+1}} \right] R_t$$
$$1 = R_t^* \mathbb{E}_t \left[\frac{\Lambda_{t,t+1}^R}{\Pi_{t,t+1}} \Pi_{t,t+1}^{\mathcal{E}} \right]$$

A.2 Households: Consumption, Investment and Export Demand

$$S_t = \frac{P_{F,t}}{P_{H,t}} \tag{A.1}$$
$$\frac{P_t}{P_{H,t}} = \left(w_C + (1-w_C) S_t^{1-\mu_C} \right)^{\frac{1}{1-\mu_C}} \tag{A.2}$$
$$\frac{P_t}{P_{F,t}} = \left(w_C S_t^{\mu_C - 1} + (1-w_C) \right)^{\frac{1}{1-\mu_C}} \tag{A.3}$$
$$\Pi_{t-1,t} = \left[w_C \left(\Pi_{H,t-1,t} \frac{P_{H,t-1}}{P_{t-1}} \right)^{1-\mu_C} + (1-w_C) \left(\Pi_{F,t-1,t} \frac{P_{F,t-1}}{P_{t-1}} \right)^{1-\mu_C} \right]^{\frac{1}{1-\mu_C}} \tag{A.4}$$
$$C_{H,t} = w_C \left(\frac{P_{H,t}}{P_t} \right)^{-\mu_C} C_t \tag{A.5}$$
$$C_{F,t} = (1-w_C) \left(\frac{P_{F,t}}{P_t} \right)^{-\mu_C} C_t \tag{A.6}$$

$$C^*_{H,t} = (1 - w^*_C) \left(\frac{P_{H,t}}{P_{F,t}} \right)^{-\mu^*_C} C^*_t \tag{A.7}$$

A.3 Firms

$$W_t = \frac{Y_t}{N_t} MC_t \frac{P_{H,t}}{P_t} \tag{A.8}$$

$$Y_t^W = A_t N_t$$
$$Y_t = \frac{Y_t^W - F}{\Delta_t}$$
$$1 = \xi \left(\Pi_{H,t-1,t} \right)^{\zeta - 1} + (1 - \xi) \left(\frac{JJ_t}{J_t} \right)^{1-\zeta}$$
$$\frac{P^0_{H,t}}{P_{H,t}} = \frac{JJ_t}{J_t}$$
$$\Delta_t = \xi \left(\Pi_{H,t-1,t} \right)^{\zeta} \Delta_{t-1} + (1 - \xi) \left(\frac{JJ_t}{J_t} \right)^{-\zeta}$$
$$JJ_t = \frac{\zeta}{\zeta - 1} Y_t MS_t MC_t + \xi \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\Pi_{H,t,t+1} \right)^{\zeta+1} \left(\Pi_{t,t+1} \right)^{-1} JJ_{t+1} \right]$$
$$J_t = Y_t + \xi \mathbb{E}_t \left[\Lambda_{t,t+1} \frac{\left(\Pi_{H,t,t+1} \right)^{\zeta}}{\Pi_{t,t+1}} J_{t+1} \right]$$
$$MC_t = \frac{P_t^W}{P_{H,t}}$$

A.4 Market Clearing

$$Y_t = C_{H,t} + C^*_{H_t}$$
$$TB_t = \frac{P_{H,t}}{P_t} Y_t - C_t$$
$$P_t^{B^*} B_{F,t} = \frac{\Pi^{\mathcal{E}}_{t-1,t}}{\Pi_{t-1,t}} B_{F,t-1} + TB_t$$

A.5 Deterministic Zero-Growth Steady State

In a non-zero-net inflation steady state given $B_F = \bar{B}_F$, $\Pi = \Pi_H = \Pi_F = \Pi^*$, with appropriate choice of units such that $\frac{P_H}{P} = \frac{P_F}{P} = \frac{P^O}{P} = 1$ we have:

$$\Pi^{\mathcal{E}} = \frac{\Pi}{\Pi^*}$$
$$N = \overline{N}$$
$$N^C = 1$$
$$N^R = \frac{N - \lambda N^C}{1 - \lambda}$$
$$S = 1$$
$$\Lambda^R = \beta$$
$$R = \frac{\Pi}{\beta}$$
$$R^* = \frac{\Pi^*}{\beta^*}$$
$$\frac{JJ}{J} = \left(\frac{1 - \xi \left(\Pi\right)^{\zeta - 1}}{1 - \xi} \right)^{\frac{1}{1-\zeta}}$$
$$MC = \frac{JJ}{J} \frac{\zeta - 1}{\zeta} \frac{1 - \xi\beta \left(\Pi_H\right)^{\zeta}}{1 - \xi\beta \left(\Pi\right)^{\zeta - 1}}$$
$$\Delta = \frac{(1 - \xi) \left(\frac{JJ}{J}\right)^{-\zeta}}{1 - \xi \left(\Pi_H\right)^{\zeta}}$$
$$C^C = N^C W$$
$$Y^W = N$$
$$Y = \frac{Y^W - F}{\Delta}$$
$$W = \frac{Y}{N} MC$$
$$B_F = \bar{B}_F$$
$$TB = \left(\frac{1}{R^*} - \frac{\Pi^S}{\Pi} \right) B_F$$
$$C = Y - TB$$
$$C_H = w_C C$$

$$C_F = (1 - w_C)C$$

$$N^R = \frac{1}{1-\lambda}N - \frac{\lambda}{1-\lambda}N^C$$

$$C^R = \frac{1}{1-\lambda}C - \frac{\lambda}{1-\lambda}C^C$$

$$\frac{P_H^0}{P_H} = \frac{J}{JJ}$$

$$JJ = \frac{\frac{\zeta}{\zeta-1}YMC}{1 - \xi\beta\left(\Pi\right)^{\zeta}}$$

$$J = \frac{Y}{1 - \xi\beta\left(\Pi\right)^{\zeta-1}}$$

$$EX = Y - C_H$$

$$C_H^* = EX$$

B Equilibrium Conditions for Model with Capital in Production and Incomplete Asset Markets

B.1 Households: Aggregate Consumption and Labour

$$C_t = \lambda C_t^C + (1-\lambda)C_t^R$$

$$N_t = \lambda N_t^C + (1-\lambda)N_t^R$$

$$\frac{(N_t^C)^{\varphi}}{(C_t^C)^{-\sigma}} = W_t$$

$$\frac{(N_t^R)^{\varphi}}{(C_t^R)^{-\sigma}} = W_t$$

$$C_t^C = W_t N_t^C$$

$$1 = \mathbb{E}_t\left[\frac{\Lambda_{t,t+1}^R}{\Pi_{t,t+1}}\right]R_t$$

$$1 = R_t^*\phi\left(\frac{\mathcal{E}_t B_{F,t}^*}{P_{H,t}Y_t}\right)\mathbb{E}_t\left[\frac{\Lambda_{t,t+1}^R}{\Pi_{t,t+1}}\Pi_{t,t+1}^{\mathcal{E}}\right]$$

B.2 Households: Consumption, Investment and Export Demand

$$S_t = \frac{P_{F,t}}{P_{H,t}} \tag{B.1}$$

$$\frac{P_t}{P_{H,t}} = \left(w_C + (1-w_C)S_t^{1-\mu_C}\right)^{\frac{1}{1-\mu_C}} \tag{B.2}$$

$$\frac{P_t}{P_{F,t}} = \left(w_C S_t^{\mu_C-1} + (1-w_C)\right)^{\frac{1}{1-\mu_C}} \tag{B.3}$$

$$\frac{P_t^I}{P_{H,t}} = \left(w_I + (1-w_I)S_t^{1-\mu_I}\right)^{\frac{1}{1-\mu_I}} \tag{B.4}$$

$$\frac{P_t^I}{P_{F,t}} = \left(w_I S_t^{\mu_I-1} + (1-w_I)\right)^{\frac{1}{1-\mu_I}} \tag{B.5}$$

$$\Pi_{t-1,t} = \left[w_C\left(\Pi_{H,t-1,t}\frac{P_{H,t-1}}{P_{t-1}}\right)^{1-\mu_C} + (1-w_C)\left(\Pi_{F,t-1,t}\frac{P_{F,t-1}}{P_{t-1}}\right)^{1-\mu_C}\right]^{\frac{1}{1-\mu_C}} \tag{B.6}$$

$$C_{H,t} = w_C\left(\frac{P_{H,t}}{P_t}\right)^{-\mu_C} C_t \tag{B.7}$$

$$C_{F,t} = (1-w_C)\left(\frac{P_{F,t}}{P_t}\right)^{-\mu_C} C_t \tag{B.8}$$

$$C_{H,t}^* = (1-w_C^*)\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\mu_C^*} C_t^* \tag{B.9}$$

$$I_{H,t} = w_I\left(\frac{P_{H,t}}{P_t^I}\right)^{-\mu_I} I_t \tag{B.10}$$

$$I_{F,t} = (1-w_I)\left(\frac{P_{F,t}}{P_t^I}\right)^{-\mu_I} I_t \tag{B.11}$$

$$I_{H,t}^* = (1-w_I^*)\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\mu_I^*} I_t^* \tag{B.12}$$

B.3 Firms

$$Z_t = \alpha\frac{Y_t}{K_{t-1}}MC_t\frac{P_{H,t}}{P_t} \tag{B.13}$$

$$W_t = (1-\alpha)\frac{Y_t}{N_t}MC_t\frac{P_{H,t}}{P_t} \tag{B.14}$$

$$R_t^k = \frac{Z_t + (1-\delta_K)Q_t}{Q_{t-1}}$$
$$Y_t^W = (A_t N_t)^{1-\alpha} K_{t-1}^{\alpha}$$
$$Y_t = \frac{Y_t^W - F}{\Delta_t}$$
$$1 = \xi \left(\Pi_{H,t-1,t}\right)^{\zeta-1} + (1-\xi)\left(\frac{JJ_t}{J_t}\right)^{1-\zeta}$$
$$\Delta_t = \xi \left(\Pi_{H,t-1,t}\right)^{\zeta} \Delta_{t-1} + (1-\xi)\left(\frac{JJ_t}{J_t}\right)^{-\zeta}$$
$$\frac{P_{H,t}^0}{P_{H,t}} = \frac{JJ_t}{J_t}$$
$$JJ_t = \frac{\zeta}{\zeta-1} Y_t MS_t MC_t + \xi \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\Pi_{H,t,t+1}\right)^{\zeta+1} \left(\Pi_{t,t+1}\right)^{-1} JJ_{t+1}\right]$$
$$J_t = Y_t + \xi \mathbb{E}_t \left[\Lambda_{t,t+1} \frac{\left(\Pi_{H,t,t+1}\right)^{\zeta}}{\Pi_{t,t+1}} J_{t+1}\right] MC_t = \frac{P_t^W}{P_{H,t}}$$
$$K_t = (1-\delta_K)K_{t-1} + (1-\mathcal{S}(X_t))I_t$$
$$Q_t(1-\mathcal{S}(X_t) - X_t\mathcal{S}'(X_t)) + \mathbb{E}_t \left[\Lambda_{t,t+1}^R \, Q_{t+1}\mathcal{S}'(X_{t+1})\frac{I_{t+1}^2}{I_t^2}\right] = \frac{P_t^I}{P_t}$$

B.4 Market Clearing

$$Y_t = C_{H,t} + C_{H,t}^* + I_{H,t} + I_{H,t}^* + G_t$$
$$EX_t = C_{H,t}^* + I_{H,t}^*$$
$$TB_t = \frac{P_{H,t}}{P_t} Y_t - C_t - \frac{P_{I,t}}{P_t} I_t - \frac{P_{H,t}}{P_t} G_t$$
$$P_t^{B*} B_{F,t} = \frac{\Pi_{t-1,t}^{\mathcal{E}}}{\Pi_{t-1,t}} B_{F,t-1} + TB_t$$

B.5 Deterministic Zero-Growth Steady State

In a non-zero-net inflation steady state and constant Q_t given $\Pi = \Pi_H = \Pi_F = \Pi^*$, with appropriate choice of units such that $\frac{P_H}{P} = \frac{P_F}{P} = \frac{P^I}{P} = \frac{P^O}{P} = 1$ we have:

$$Q = 1$$

$$\Pi^{\mathcal{E}} = \frac{\Pi}{\Pi^*}$$

$$N = \overline{N}$$

$$N^C = 1$$

$$N^R = \frac{N - \lambda N^C}{1 - \lambda}$$

$$S = 1$$

$$\Lambda^R = \beta$$

$$Q = 1$$

$$R = \frac{\Pi}{\beta}$$

$$R^* = \frac{\Pi^*}{\beta^*}$$

$$\phi = \frac{R}{R^* \Pi^{\mathcal{E}}} = \frac{\Pi}{\Pi^* \Pi^{\mathcal{E}}} \frac{\beta^*}{\beta} = \frac{\beta^*}{\beta}$$

$$R^k = R$$

$$\frac{JJ}{J} = \left(\frac{1 - \xi \left(\Pi\right)^{\zeta - 1}}{1 - \xi} \right)^{\frac{1}{1-\zeta}}$$

$$MC = \frac{JJ}{J} \frac{\zeta - 1}{\zeta} \frac{1 - \xi\beta \left(\Pi_H\right)^{\zeta}}{1 - \xi\beta \left(\Pi\right)^{\zeta - 1}}$$

$$\Delta = \frac{\left(1 - \xi\right) \left(\frac{JJ}{J}\right)^{-\zeta}}{1 - \xi \left(\Pi_H\right)^{\zeta}}$$

$$\frac{N}{K} = \left(\frac{\frac{R^K}{\Pi} - \left(1 - \delta_K\right)}{MC\alpha \left(1 - \tau^k\right)} \right)^{1/(1-\alpha)}$$

$$W = \left(1 - \alpha\right) \left(\frac{N}{K} \right)^{\alpha} MC$$

$$C^C = N^C W$$

$$K = \frac{N}{\frac{N}{K}}$$

$$Y^W = K^{\alpha} N^{1-\alpha}$$

$$I = \delta_K K$$

$$Y = \frac{Y^W - F}{\Delta}$$

$$
B_F = -\frac{Y \log(\phi)}{\phi_B} \geq 0 \text{ iff } \beta \geq \beta^*
$$

$$
TB = \left(\frac{1}{\phi R^*} - \frac{\Pi^S}{\Pi} \right) B_F
$$

$$
G = g_y Y
$$

$$
C = Y - I - G - TB
$$

$$
C_H = w_C C
$$

$$
C_F = (1 - w_C) C
$$

$$
N^R = \frac{1}{1-\lambda} N - \frac{\lambda}{1-\lambda} N^C
$$

$$
I_H = w_I I
$$

$$
I_F = (1 - w_I) I
$$

$$
C^R = \frac{1}{1-\lambda} C - \frac{\lambda}{1-\lambda} C^C
$$

$$
\frac{P_H^0}{P_H} = \frac{J}{JJ}
$$

$$
JJ = \frac{\frac{\zeta}{\zeta-1} YMC}{1 - \xi\beta \left(\Pi\right)^{\zeta}}
$$

$$
J = \frac{Y}{1 - \xi\beta \left(\Pi\right)^{\zeta-1}}
$$

$$
EX = Y - C_H - I_H - G
$$

$$
C_H^* = EX_C = EX_C(1) = cs_{exp} EX
$$

$$
I_H^* = EX_I = EX_I(1) = is_{exp} EX
$$

B.6 Dominant (or Local) Currency Pricing

To characterize exports being priced in a dominant currency, or indeed any foreign currency, we introduce a second retail sector for exports in which the price is set in the foreign currency.[37] The nominal price rigidity is in the foreign currency.

The objective of domestic exporter m at time t is to choose $P_{X,t}^0(m)$ to maximize discounted

[37]In general, there can be important differences between models characterized by dominant currency pricing (DCP) and local currency pricing (LCP). However, for the determinacy exercise, we only need to make sure that exports are priced in the foreign currency.

real profits:

$$\mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \frac{\Lambda_{t,t+k}}{P_{t+k}} EX_{t+k}(m) \left[\mathcal{E}_{t+k} P^0_{X,t}(m) - P_{H,t+k} MC_{t+k} \right] \tag{B.15}$$

noting real marginal costs are deflated by the wholesale producer price index $P_{H,t}$, so $MC_t \equiv P^W_t / P_{H,t}$. $\mathcal{E}_{t+k}$ is the nominal exchange rate. This is maximized is subject to demand schedules:

$$EX_{t+k}(m) = \left(\frac{P^0_{X,t}(m)}{P^*_{H,t+k}} \right)^{-\zeta} EX_{H,t+k}, \tag{B.16}$$

where $\Lambda_{t,t+k} \equiv \beta^k \frac{U_{C,t+k}}{U_{C,t}}$ is the stochastic discount factor over the interval $[t, t+k]$. Using the gross nominal depreciation of the SOE currency, $\Pi^{\mathcal{E}}_{t-1,t} \equiv \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}$, this leads to an optimal pricing condition

$$\frac{P^0_{X,t}}{P^*_{H,t}} = \frac{\zeta}{\zeta - 1} \frac{P_{H,t}}{\mathcal{E}_t P^*_{H,t}} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \Lambda_{t,t+k} \left(\Pi_{t,t+k}\right)^{-1} \left(\Pi^*_{H,t+k}\right)^{\zeta} EX_{H,t+k} \Pi_{H,t,t+k} MC_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} \xi^k \Lambda_{t,t+k} \left(\Pi_{t,t+k}\right)^{-1} \left(\Pi^*_{H,t+k}\right)^{\zeta} \Pi^{\mathcal{E}}_{t,t+k} EX_{H,t+k}} \tag{B.17}$$

Total output in the two retail sectors, domestically produced and sold goods, $Y_{H,t}$, and export, EX_t, are subject to the effects of price dispersion $\Delta_t \equiv \int_0^1 \left(\frac{P_{H,t}(m)}{P_{H,t}} \right)^{-\varsigma} dm \geq 1$ and $\Delta_{X,t} \equiv \int_0^1 \left(\frac{P_{X,t}(m)}{P^*_{H,t}} \right)^{-\varsigma} dm \geq 1$. Aggregate output Y_t is given by:

$$Y_t = \frac{Y_t - EX_t}{Y_t} \frac{A_t N_t - F}{\Delta_t} + \frac{EX_t}{Y_t} \frac{A_t N_t - F}{\Delta_{X,t}}, \tag{B.18}$$

B.7 Calibration of Model Parameters

The baseline and alternative calibrations of the model used to produce numerical results are given in Table 2.

Parameter	Description	Value
β $(= \beta^*)$	Home (foreign) discount factor	0.99
Π $(= \Pi^*)$	Steady-state home (foreign) inflation rate	$1, 1.04^{1/4}, 1.06^{1/4}$
σ	Relative risk aversion	2
φ	Inverse of the Frisch elasticity of labour supply	2
α	Income share of capital	$0, 0.3$
δ_K	Depreciation rate of capital	0.025
ϕ_I	Investment adjustment costs	10
ξ	Degree of price stickiness	0.75
$1 - \lambda$	Degree of LAMP	$\lambda \in [0, 1]$
μ_C $(= \mu_C^*)$, μ_I $(= \mu_I^*)$	Elasticity of substitution between home and foreign goods	0.62
$1 - w_C$, $1 - w_I$	Degree of trade openness	0.4
ϕ_B	Bond adjustment costs	0.001
ζ	Elasticity of substitution of differentiated goods	7
g_Y	Government spending share of output	0.1

Table 2: Parameter Values Used in the Numerical Analysis of Determinacy

C Equilibrium Determinacy: Derivations and Proofs

C.1 Derivation of the Minimal Form of the Dynamic System

The model is linearized around a zero-growth, zero-inflation steady state so $\Pi = 1$ and prices $P = P_H = P_F = P^* = 1$. Then by definition the steady state terms of trade and real exchange rate are $\mathcal{E} = Q = 1$. As discussed in the main text, we assume an equitable steady state.[38] All lower-case variables denote percentage deviations from the steady state. All shocks are set equal to zero.

Aggregate demand:

$$y_t = w_C c_t + (1 - w_C)(c_t^R + \omega s_t), \tag{C.1}$$

[38]Therefore, we need either the zero-profit condition, $F/Y = 1/\zeta$, or the subsidy scheme that supports the welfare-relevant choice of the output gap x_t.

where $\omega \equiv \frac{w_C(\sigma\mu_C - 1) + \sigma\mu_C^*}{\sigma} = \frac{\sigma\mu_C(1+w_C) - w_C}{\sigma}$, if $\mu_C = \mu_C^*$.

Aggregate supply:

$$\pi_{H,t} = \beta\pi_{H,t+1} + \Psi mc_t, \tag{C.2}$$

$$mc_t = w_t + (1 - w_C)s_t, \tag{C.3}$$

$$y_t = \lambda\left(1 + \frac{1}{\zeta}\right) n_t^R + (1-\lambda)\left(1 + \frac{1}{\zeta}\right) n_t^C, \tag{C.4}$$

where $\Psi \equiv \frac{(1-\xi)(1-\beta\xi)}{\xi} > 0$ and

$$\pi_t = \pi_{H,t} + (1 - w_C)(s_t - s_{t-1}). \tag{C.5}$$

Household optimality conditions:

$$w_t = \varphi n_t^R + \sigma c_t^R, \tag{C.6}$$

$$w_t = \varphi n_t^C + \sigma c_t^C, \tag{C.7}$$

$$c_t = \lambda c_t^R + (1-\lambda)c_t^C, \tag{C.8}$$

$$c_t^C = w_t + n_t^C, \tag{C.9}$$

$$s_t = \frac{\sigma}{w_C} c_t^R, \tag{C.10}$$

$$c_t^R = c_{t+1}^R - \frac{1}{\sigma}(r_t - \pi_{t+1}). \tag{C.11}$$

Combining (C.5), (C.10), and (C.11), we obtain:

$$\begin{aligned} \pi_t &= \pi_{H,t} + (1-w_C)\frac{\sigma}{w_C}(c_t^R - c_{t-1}^R) \\ &= \pi_{H,t} + (1-w_C)\frac{1}{w_C}(r_{t-1} - \pi_t). \end{aligned} \tag{C.12}$$

It follows that

$$r_t - \pi_{t+1} = r_t - \pi_{H,t+1} - (1-w_C)\frac{1}{w_C}(r_t - \pi_{t+1})$$

from which

$$r_t - \pi_{t+1} = w_C(r_t - \pi_{H,t+1}). \tag{C.13}$$

The intertemporal IS equation (C.11) can be expressed as:

$$c_{t+1}^R + \frac{w_C}{\sigma}\pi_{H,t+1} - \frac{w_C}{\sigma} r_t = c_t^R. \tag{C.14}$$

Using (C.6) to eliminate w_t and (C.10) to eliminate s_t, equations (C.3), (C.7), and (C.9) become:

$$mc_t = \varphi n_t^R + \frac{\sigma}{w_C} c_t^R, \tag{C.15}$$

$$\varphi n_t^C = \varphi n_t^R + \sigma c_t^R - \sigma c_t^C, \tag{C.16}$$

$$c_t^C = \varphi n_t^R + \sigma c_t^R + n_t^C. \tag{C.17}$$

Using (C.8) and (C.10) to eliminate c_t and s_t from (C.1) yields:

$$y_t = w_C \lambda c_t^R + w_C(1-\lambda) c_t^C + (1-w_C)\left[1 + \frac{\omega\sigma}{w_C}\right] c_t^R, \tag{C.18}$$

and rearranging (C.4)

$$n_t^R = \frac{1}{\lambda\left(1+\frac{1}{\zeta}\right)} y_t - \frac{(1-\lambda)}{\lambda} n_t^C. \tag{C.19}$$

Combining (C.16) and (C.17) gives:

$$n_t^C = \frac{\varphi(1-\sigma)}{\varphi+\sigma} n_t^R + \frac{\sigma(1-\sigma)}{\varphi+\sigma} c_t^R. \tag{C.20}$$

Combining (C.17), (C.18), and (C.20) yields:

$$y_t = w_C \lambda c_t^R + w_C(1-\lambda)\frac{(1+\varphi)}{\varphi+\sigma}\left[\varphi n_t^R + \sigma c_t^R\right] + (1-w_C)\left[1+\frac{\omega\sigma}{w_C}\right] c_t^R. \tag{C.21}$$

Combining (C.19), (C.20), and (C.21) gives:

$$n_t^R = \left[\frac{w_C\lambda(\varphi+\sigma) + (1-w_C)(\varphi+\sigma)\left[1+\frac{\omega\sigma}{w_C}\right] + (1-\lambda)\sigma\left[w_C(1+\varphi) + (\sigma-1)\left(1+\frac{1}{\zeta}\right)\right]}{\lambda(\varphi+\sigma)\left(1+\frac{1}{\zeta}\right) - (1-\lambda)\varphi\left[w_C(1+\varphi) + (\sigma-1)\left(1+\frac{1}{\zeta}\right)\right]}\right] c_t^R. \tag{C.22}$$

Finally, combining (C.2), (C.15), and (C.19) – (C.22) results in

$$\beta\pi_{H,t+1} \quad = \quad \pi_{H,t} - \Psi\Upsilon c_t^R \tag{C.23}$$

$$y_t \quad = \quad \Xi c_t^R \tag{C.24}$$

where

$$\Upsilon = \frac{\sigma}{w_C} + \frac{\varphi\lambda w_C(\varphi+\sigma) + \varphi(1-w_C)(\varphi+\sigma)\left[1+\frac{\omega\sigma}{w_C}\right] + \varphi\sigma(1-\lambda)\left[w_C(1+\varphi)+(\sigma-1)\left(1+\frac{1}{\zeta}\right)\right]}{\lambda(\varphi+\sigma)\left(1+\frac{1}{\zeta}\right) - (1-\lambda)\varphi\left[w_C(1+\varphi)+(\sigma-1)\left(1+\frac{1}{\zeta}\right)\right]},$$

$$\Rightarrow \Upsilon = \frac{\sigma(1-w_C)}{w_C} + \frac{\lambda(\varphi+\sigma)\left[w_C\varphi+\sigma\left(1+\frac{1}{\zeta}\right)\right] + \varphi(1-w_C)(\varphi+\sigma)\left[1+\frac{\omega\sigma}{w_C}\right]}{\lambda(\varphi+\sigma)\left(1+\frac{1}{\zeta}\right) - (1-\lambda)\varphi\left[w_C(1+\varphi)+(\sigma-1)\left(1+\frac{1}{\zeta}\right)\right]}. \tag{C.25}$$

and

$$\Xi \equiv w_C\lambda + (1-w_C)\left[1+\frac{\omega\sigma}{w_C}\right] + w_C(1-\lambda)\frac{(1+\varphi)}{\varphi+\sigma}\left(\Upsilon - \frac{\sigma}{w_C} + \sigma\right), \tag{C.26}$$

The interest-rate rule is given by:

$$r_t = \rho_r r_{t-1} + \theta_\pi \pi_{t+1}. \tag{C.27}$$

The dynamic system given by (C.14), (C.23), and (C.27) can be expressed as:

$$\mathbf{z}_{t+1} = \mathbf{A}\mathbf{z}_t, \quad \mathbf{z}_t = \left[c_t^R \ \pi_{H,t} \ r_{t-1}\right]',$$

$$\mathbf{A} \equiv \begin{bmatrix} 1 - \frac{\Psi w_C \Upsilon(\theta_\pi - 1)}{\beta\sigma[1-(1-w_C)\theta_\pi]} & \frac{w_C(\theta_\pi-1)}{\beta\sigma[1-(1-w_C)\theta_\pi]} & \frac{\rho_r w_C}{\sigma[1-(1-w_C)\theta_\pi]} \\ -\frac{\Psi\Upsilon}{\beta} & \frac{1}{\beta} & 0 \\ -\frac{\Psi w_C \Upsilon \theta_\pi}{\beta[1-(1-w_C)\theta_\pi]} & \frac{w_C\theta_\pi}{\beta[1-(1-w_C)\theta_\pi]} & \frac{\rho_r}{[1-(1-w_C)\theta_\pi]} \end{bmatrix}.$$

C.2 The NKIS and NKPC Equations with Assumption 1 Restrictions

From (C.14) with $\sigma = 1$:

$$y_t = y_{t+1} - \Xi w_C\left(r_t - \pi_{H,t+1}\right). \tag{C.28}$$

where with $\sigma = \omega = 1$, (C.26) becomes

$$\Xi \quad = \quad w_C\lambda + (1-w_C)\left[1+\frac{1}{w_C}\right] + w_C(1-\lambda)\left(\Upsilon - \frac{1}{w_C} + 1\right)$$

$$= \frac{\lambda}{w_C(\lambda - (1-\lambda)w_C\varphi)} \tag{C.29}$$

In the flexi-price case, $\pi_{H,t} = 0$, and hence:

$$y_t^n = y_{t+1}^n - \Xi w_C r_t^n. \tag{C.30}$$

Hence in terms of the output gap $x_t = y_t - y_t^n$ we have

$$x_t = x_{t+1} - \Xi w_C \left(r_t - \pi_{H,t+1} - r_t^n\right). \tag{C.31}$$

Next we turn to the NKPC which from (C.23) and (C.24) is given by

$$\beta\pi_{H,t+1} = \pi_{H,t} - \frac{\Psi\Upsilon}{\Xi} y_t \tag{C.32}$$

where with $\sigma = \omega = 1$ and $\zeta \to \infty$, (C.25) becomes (with a little algebra)

$$\begin{aligned} \Upsilon &= \frac{1-w_C}{w_C} + \frac{\lambda(w_C\varphi + 1) + (1-w_C)\varphi\left(1 + \frac{1}{w_C}\right)}{\lambda - (1-\lambda)w_C\varphi} \\ &= \frac{(1+w_C\varphi)\lambda + (1-w_C)\varphi}{w_C(\lambda - (1-\lambda)w_C\varphi)} \end{aligned} \tag{C.33}$$

Hence

$$\Xi = \frac{\lambda}{w_C(\lambda - (1-\lambda)w_C\varphi)} \tag{C.34}$$

and the slope of the NKPC (C.32) can be written $\kappa \equiv \frac{\Psi\Upsilon}{\Xi} = \Psi\frac{(1+w_C\varphi)\lambda + (1-w_C)\varphi}{\lambda}$.

C.3 Proof of Proposition 1

Differentiating (3.6) we obtain

$$\frac{d\lambda^*}{dw_C} = \frac{\varphi(1+\varphi)(\varphi+\sigma)\left(1+\frac{1}{\zeta}\right)}{\left[\varphi\left[w_C(1+\varphi) + (\sigma-1)\left(1+\frac{1}{\zeta}\right)\right] + (\varphi+\sigma)\left(1+\frac{1}{\zeta}\right)\right]^2}.$$

Hence $\frac{d\lambda^*}{dw_C} > 0$ from which the proposition is proved. □

C.4 Proof of Proposition 2

The necessary and sufficient conditions for equilibrium determinacy are as follows:

Case IA: If $\Upsilon > 0$: $\frac{1}{1-w_C} < \theta_\pi < \Gamma_1$ and $\rho_r > \left(\frac{w_C}{1-w_C}\right)\left[\frac{\Psi w_C \Upsilon}{\Psi w_C \Upsilon + 2\sigma(1+\beta)}\right]$, where

$$\Gamma_1 \equiv (1+\rho_r)\left[1 + \frac{2(1+\beta)\sigma w_C}{\Psi w_C \Upsilon + 2\sigma(1+\beta)(1-w_C)}\right];$$

Case IB: If $\Upsilon > 0$: $1-\rho_r < \theta_\pi < \min\left\{\frac{1}{1-w_C}, \Gamma_1\right\}$ and one of the following inequalities is satisfied:

$$\left|-1 - \frac{1}{\beta} - \frac{\rho_r}{[1-(1-w_C)\theta_\pi]} + \frac{\Psi w_C \Upsilon(\theta_\pi - 1)}{\beta\sigma\left[1-(1-w_C)\theta_\pi\right]}\right| > 3, \tag{C.35}$$

$$1-\beta + \frac{\rho_r}{1-(1-w_C)\theta_\pi}\left[\frac{\rho_r(1-\beta)}{\beta[1-(1-w_C)\theta_\pi]} + \beta - \frac{1}{\beta} + \frac{\Psi w_C \Upsilon}{\sigma}\left(1 + \frac{\theta_\pi - 1}{\beta[1-(1-w_C)\theta_\pi]}\right)\right] > 0; \tag{C.36}$$

Case IIA: If $\Upsilon < -\frac{2\sigma(1+\beta)}{\Psi w_C}$: and one of the following is satisfied:

(i) $1-\rho_r < \theta_\pi < \min\left\{\frac{1}{1-w_C},\ \Gamma_1\right\}$,

(ii) $\Gamma_1 < \theta_\pi < 1-\rho_r$, $(1-\rho_r)(1+\beta)\sigma w_C + \left[2\sigma(1+\beta) + \Psi w_C \Upsilon\right]\rho_r > 0$ and one of the inequalities given by (C.35)–(C.36) holds,

(iii) $\frac{1}{1-w_C} < \theta_\pi < \Gamma_1,\ \rho_r > \left(\frac{w_C}{1-w_C}\right)\left[\frac{\Psi w_C \Upsilon}{\Psi w_C \Upsilon + 2\sigma(1+\beta)}\right]$, and one of the inequalities given by (C.35)–(C.36) holds;

Case IIB: If $-\frac{2\sigma(1+\beta)}{\Psi w_C} < \Upsilon < -\frac{2\sigma(1+\beta)(1-w_C)}{\Psi w_C}$: $\theta_\pi < 1-\rho_r$, and one of the inequalities given by (C.35)–(C.36) is satisfied;

Case IIC: If $-\frac{2\sigma(1+\beta)(1-w_C)}{\Psi w_C} < \Upsilon < 0$: one of the inequalities given by (C.35)–(C.36) is satisfied, and either $\theta_\pi < 1-\rho_r$ or $\theta_\pi > \max\left\{\frac{1}{1-w_C}, \Gamma_1\right\}$.

These arise from the following derivation. The minimum state-space representation of the model is $\mathbf{z}_{t+1} = \mathbf{A}\mathbf{z}_t$ where $\mathbf{A}$ is given by (C.1). The three eigenvalues of $\mathbf{A}$ are solutions to the cubic equation $r^3 + a_2 r^2 + a_1 r + a_0 = 0$, where $a_2 = -1 - \frac{1}{\beta} - \frac{\rho_r}{[1-(1-w_C)\theta_\pi]} + \frac{\Psi w_C \Upsilon(\theta_\pi - 1)}{\beta\sigma[1-(1-w_C)\theta_\pi]}$, $a_1 = \frac{1}{\beta} + \frac{(1+\beta)\rho_r}{\beta[1-(1-w_C)\theta_\pi]} + \frac{\Psi w_C \Upsilon \rho_r}{\beta\sigma[1-(1-w_C)\theta_\pi]}$, and $a_0 = -\frac{\rho_r}{\beta[1-(1-w_C)\theta_\pi]}$. With one predetermined variable r_{t-1}, determinacy requires that one eigenvalue is inside the unit circle and two eigenvalues are outside the unit circle. By Proposition C.2 of Woodford (2003), this is the case if and only if either of the following two cases is satisfied: Case I: $1+a_2+a_1+a_0<0$, $-1+a_2-a_1+a_0>0$, Case II: $1+a_2+a_1+a_0>0$, $-1+a_2-a_1+a_0<0$, and either $|a_2|>3$ or $a_0^2-a_0a_2+a_1-1>0$.

For Case I, the two inequalities reduce to:

$$\frac{\Psi w_C \Upsilon(\theta_\pi - 1 + \rho_r)}{\beta\sigma\left[1-(1-w_C)\theta_\pi\right]} < 0, \tag{C.37}$$

$$-2(1+\beta)\left(1+\frac{\rho_r}{\left[1-(1-w_C)\theta_\pi\right]}\right)+\frac{\Psi w_C \Upsilon(\theta_\pi - 1 - \rho_r)}{\sigma\left[1-(1-w_C)\theta_\pi\right]} > 0. \tag{C.38}$$

First assume that $\Upsilon>0$. Condition (C.37) requires either (i) $0<\theta_\pi<\min\left\{1-\rho_r, \frac{1}{1-w_C}\right\}$ or (ii) $\max\left\{1-\rho_r, \frac{1}{1-w_C}\right\}<\theta_\pi$. By inspection, (C.38) can never be satisfied under (i). For (ii), first note that the lower-bound $1-\rho_r$ is redundant since $1-\rho_r<\frac{1}{1-w_C}$ and (C.38) requires:

$$\theta_\pi < \frac{(1+\rho_r)\left[\Psi w_C\Upsilon + 2\sigma(1+\beta)\right]}{\Psi w_C \Upsilon + 2\sigma(1+\beta)(1-w_C)}. \tag{C.39}$$

For the determinacy region to be non-empty requires $\left(\frac{w_C}{1-w_C}\right)\left[\frac{\Psi w_C\Upsilon}{\Psi w_C\Upsilon+2\sigma(1+\beta)}\right]<\rho_r$. Now assume that $\Upsilon<0$. Condition (C.37) requires that $1-\rho_r<\theta_\pi<\frac{1}{1-w_C}$ and (C.38) requires $\theta_\pi\left[\Psi w_C\Upsilon+2\sigma(1+\beta)(1-w_C)\right]>(1+\rho_r)\left[\Psi w_C\Upsilon+2\sigma(1+\beta)\right]$. The latter can only be satisfied if $\Psi w_C\Upsilon+2\sigma(1+\beta)(1-w_C)<0$ and $\Psi w_C\Upsilon+2\sigma(1+\beta)<0$, which requires an additional upper-bound on θ_π given by (C.39).

For Case II, the first two inequalities reduce to:

$$\frac{\Psi w_C \Upsilon(\theta_\pi - 1 + \rho_r)}{\beta\sigma\left[1-(1-w_C)\theta_\pi\right]} > 0, \tag{C.40}$$

$$-2(1+\beta)\left(1+\frac{\rho_r}{\left[1-(1-w_C)\theta_\pi\right]}\right)+\frac{\Psi w_C \Upsilon(\theta_\pi - 1 - \rho_r)}{\sigma\left[1-(1-w_C)\theta_\pi\right]} < 0. \tag{C.41}$$

First assume that $\Upsilon>0$. Equation (C.40) requires that $1-\rho_r<\theta_\pi<\frac{1}{1-w_C}$ and (C.41) requires the upper-bound on θ_π given by (C.39). The remaining inequalities give (C.35) and (C.36). Now assume that $\Upsilon<0$. Condition (C.40) requires either (i) $0<\theta_\pi<\min\left\{1-\rho_r, \frac{1}{1-w_C}\right\}$ or (ii) $\max\left\{1-\rho_r, \frac{1}{1-w_C}\right\}<\theta_\pi$. For (i), first note that $1-\rho_r<\frac{1}{1-w_C}$, and (C.41) requires $\theta_\pi\left[\Psi w_C\Upsilon+2\sigma(1+\beta)(1-w_C)\right]<(1+\rho_r)\left[\Psi w_C\Upsilon+2\sigma(1+\beta)\right]$, which is always satisfied if $\Psi w_C \Upsilon + 2\sigma(1+\beta)(1-w_C) < 0$ and $\Psi w_C \Upsilon + 2\sigma(1+\beta) > 0$. If $\Psi w_C \Upsilon + 2\sigma(1+\beta)(1-w_C) > 0$ and $\Psi w_C \Upsilon + 2\sigma(1+\beta) > 0$, the upper-bound (C.39) is redundant. If $\Psi w_C \Upsilon + 2\sigma(1+\beta)(1-w_C) < 0$ and $\Psi w_C \Upsilon + 2\sigma(1+\beta) < 0$, the following lower-bound on θ_π is needed:

$$\theta_\pi > \frac{(1+\rho_r)\left[\Psi w_C \Upsilon + 2\sigma(1+\beta)\right]}{\Psi w_C \Upsilon + 2\sigma(1+\beta)(1-w_C)}. \tag{C.42}$$

For the region to be non-empty requires $(1-\rho_r)(1+\beta)\sigma w_C + \left[2\sigma(1+\beta) + w_C \Psi \Upsilon\right]\rho_r > 0$. The remaining inequalities give (C.35) and (C.36). For (ii), the lower-bound $1-\rho_r$ is redundant and (C.41) requires $\theta_\pi\left[\Psi w_C \Upsilon + 2\sigma(1+\beta)(1-w_C)\right] > (1+\rho_r)\left[\Psi w_C \Upsilon + 2\sigma(1+\beta)\right]$, which can never be satisfied if $\Psi w_C \Upsilon + 2\sigma(1+\beta)(1-w_C) < 0$ and $\Psi w_C \Upsilon + 2\sigma(1+\beta) > 0$. If $\Psi w_C \Upsilon + 2\sigma(1+\beta)(1-w_C) > 0$ and $\Psi w_C \Upsilon + 2\sigma(1+\beta) > 0$, an additional lower-bound on θ_π given by (C.41) is required. If $\Psi w_C \Upsilon + 2\sigma(1+\beta)(1-w_C) < 0$ and $\Psi w_C \Upsilon + 2\sigma(1+\beta) < 0$, requires the upper-bound on θ_π given by (C.39). For the determinacy region to be non-empty requires $\left(\frac{w_C}{1-w_C}\right)\left[\frac{\Psi w_C \Upsilon}{\Psi w_C \Upsilon + 2\sigma(1+\beta)}\right] < \rho_r$. The remaining inequalities give (C.35) and (C.36). This completes the proof. □

C.5 Proof of Proposition 3

The necessary and sufficient conditions for equilibrium determinacy under a domestic-price inflation rule with interest-rate inertia are as follows:

$$\text{Case I:} \quad \text{If} \;\; \Upsilon > 0 : \max\{0, 1-\rho_r\} < \theta_\pi < (1+\rho_r)\left[1 + \frac{2\sigma(1+\beta)}{\Psi w_C \Upsilon}\right],$$

and one of the following inequalities is satisfied:

$$\left| -1 - \frac{1}{\beta} - \rho_r + \frac{\Psi w_C \Upsilon(\theta_\pi - 1)}{\beta\sigma} \right| > 3, \tag{C.43}$$

$$1 - \beta + \rho_r\left[\frac{\rho_r(1-\beta)}{\beta} + \beta - \frac{1}{\beta} + \frac{\Psi w_C \Upsilon}{\sigma}\left(1 + \frac{\theta_\pi - 1}{\beta}\right)\right] > 0. \tag{C.44}$$

$$\text{Case IIA:} \quad \Upsilon < 0, \quad \text{and} \quad \max\{0, 1-\rho_r\} < \theta_\pi < (1+\rho_r)\left[1 + \frac{2\sigma(1+\beta)}{\Psi w_C \Upsilon}\right].$$

$$\text{Case IIB:} \quad \Upsilon < 0, \quad (1+\rho_r)\left[1 + \frac{2\sigma(1+\beta)}{\Psi w_C \Upsilon}\right] < \theta_\pi < 1 - \rho_r,$$

and one of the inequalities given by (C.43) and (C.44) is satisfied.

These conditions arise from the following derivation. The dynamic system is given by:

$$c_{t+1}^R + \frac{w_C}{\sigma}\pi_{H,t+1} - \frac{w_C}{\sigma} r_t = c_t^R, \tag{C.45}$$

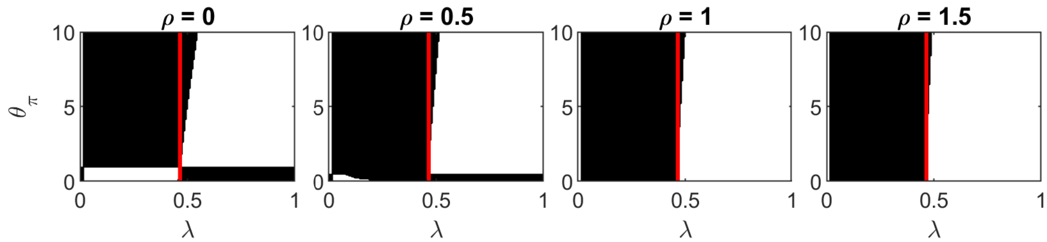
$$\beta\pi_{H,t+1} = \pi_{H,t} - \Psi\Upsilon c_t^R, \tag{C.46}$$

$$r_t = \rho_r r_{t-1} + \theta_\pi \pi_{H,t+1}, \tag{C.47}$$

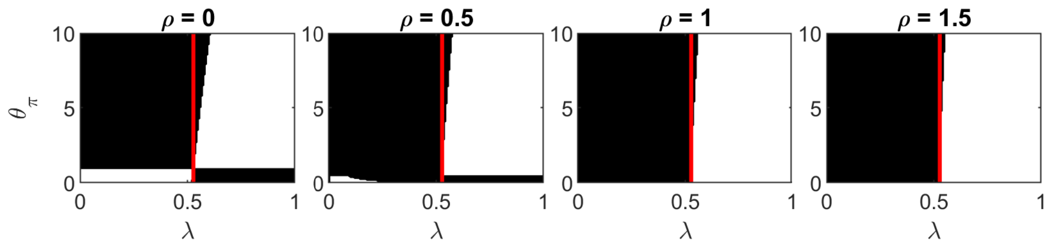
which can be expressed as:

$$\mathbf{z}_{t+1} = \mathbf{A_2}\mathbf{z}_t, \quad \mathbf{z}_t = \left[c_t^R \ \pi_{H,t} \ r_{t-1}\right]', \quad \mathbf{A_2} \equiv \begin{bmatrix} 1 - \frac{\Psi w_C \Upsilon(\theta_\pi - 1)}{\beta\sigma} & \frac{w_C(\theta_\pi - 1)}{\beta\sigma} & \frac{\rho_r w_C}{\sigma} \\ -\frac{\Psi\Upsilon}{\beta} & \frac{1}{\beta} & 0 \\ -\frac{\Psi\Upsilon\theta_\pi}{\beta} & \frac{\theta_\pi}{\beta} & \rho_r \end{bmatrix}.$$

The three eigenvalues of $\mathbf{A_2}$ are solutions to the cubic equation $r^3 + a_2 r^2 + a_1 r + a_0 = 0$, where $a_2 = -1 - \frac{1}{\beta} - \rho_r + \frac{\Psi w_C \Upsilon(\theta_\pi - 1)}{\beta\sigma}$, $a_1 = \frac{1}{\beta} + \frac{(1+\beta)\rho_r}{\beta} + \frac{\Psi w_C \Upsilon \rho_r}{\beta\sigma}$, and $a_0 = -\frac{\rho_r}{\beta}$. With one predetermined variable r_{t-1}, determinacy requires that one eigenvalue is inside the unit circle and two eigenvalues are outside the unit circle. By Proposition C.2 of Woodford (2003), this is the case if and only if either of the following two cases is satisfied: Case I: $1+a_2+a_1+a_0 < 0$, $-1+a_2-a_1+a_0 > 0$, Case II: $1+a_2+a_1+a_0 > 0$, $-1+a_2-a_1+a_0 < 0$, and either $|a_2| > 3$ or $a_0^2 - a_0 a_2 + a_1 - 1 > 0$. For Case I, the second inequality can never be satisfied with $\Upsilon > 0$. With $\Upsilon < 0$, the first inequality of Case I requires $1 - \rho_r < \theta_\pi$ and the second inequality yields $\theta_\pi < (1+\rho_r)\left[1 + \frac{2\sigma(1+\beta)}{\Psi w_C \Upsilon}\right]$. For Case II, the first inequality requires either $\Upsilon > 0$ and $\theta_\pi > 1 - \rho_r$, or $\Upsilon < 0$ and $\theta_\pi < 1 - \rho_r$. With $\Upsilon > 0$, the second inequality is automatically satisfied if $\theta_\pi < 1 + \rho_r$. Otherwise, the following upper-bound $\theta_\pi < (1+\rho_r)\left[1 + \frac{2\sigma(1+\beta)}{\Psi w_C \Upsilon}\right]$ is additionally required. With $\Upsilon < 0$, the second inequality yields $\theta_\pi > (1+\rho_r)\left[1 + \frac{2\sigma(1+\beta)}{\Psi w_C \Upsilon}\right]$. The remaining inequalities of Case II give (C.43) and (C.44). This completes the proof. The analytical conditions indicate that increasing policy inertia enlarges the region of determinacy under Case I and IIA, and shrinks the determinacy region under Case IIB. For a standard range of parameter values, Case IIA only holds for a small range of $\lambda < \lambda^*$ and the combined impact of increased inertia on the bounds of Cases IIA and IIB results in an overall reduced policy space. The effect of openness is ambiguous from these conditions, however from numerical results, we find that



(a) Open economy



(b) Closed economy

Figure 11: Determinacy regions for small model with domestic-price inflation targeting. Parameter values are $\Psi = 0.086$, $\varphi = \sigma = 2$, $\zeta = 7$, $\beta = 0.99$, and $w_C = 0.6$, $\mu_C = 0.62$ for the open economy (top panel) and $w_C = 1$ for the closed economy (bottom panel). The red vertical line gives λ^* below which IADL holds.

openness appears to enlarge the determinate policy space under SADL and shrink it under IADL for standard parameter. Determinacy regions are shown in Figure 11.

C.6 Proof of Proposition 4

The necessary and sufficient conditions for equilibrium determinacy under an interest-rate rule that reacts to future consumer-price inflation rule and contemporaneous output with interest-rate inertia are:

Case IA: $\max\left\{\frac{1}{1-w_C}, 1-\rho_r-\frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y\right\} < \theta_\pi < \Gamma_1$ and $\rho_r > \left(\frac{w_C}{1-w_C}\right)\left[\frac{\Psi w_C\Upsilon-(1-w_C)(1+\beta)\Xi\theta_y}{\Psi w_C\Upsilon+2\sigma(1+\beta)}\right]$.

Case IB: $\max\left\{0, 1-\rho_r-\frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y\right\} < \theta_\pi < \min\left\{\frac{1}{1-w_C}, \Gamma_1\right\}$ and one of the following inequalities is satisfied:

$$\left|-1-\frac{1}{\beta}-\frac{\rho_r}{[1-(1-w_C)\theta_\pi]}+\frac{\Psi w_C\Upsilon(\theta_\pi-1)}{\beta\sigma[1-(1-w_C)\theta_\pi]}-\frac{w_C\Xi\theta_y}{\sigma[1-(1-w_C)\theta_\pi]}\right|>3, \tag{C.48}$$

$$1-\beta+\frac{\rho_r}{1-(1-w_C)\theta_\pi}\frac{\rho_r(1-\beta)}{\beta[1-(1-w_C)\theta_\pi]}+\beta-\frac{1}{\beta}+\frac{\Psi w_C\Upsilon}{\sigma}\left(1+\frac{\theta_\pi-1}{\beta[1-(1-w_C)\theta_\pi]}\right)$$
$$+\frac{w_C\Xi\theta_y}{1-(1-w_C)\theta_\pi}\left[\frac{1-\rho_r-(1-w_C)\theta_\pi}{\sigma[1-(1-w_C)\theta_\pi]}\right]>0, \tag{C.49}$$

where

$$\Gamma_1\equiv(1+\rho_r)\left[1+\frac{2(1+\beta)\sigma w_C}{\Psi w_C\Upsilon+2\sigma(1+\beta)(1-w_C)}\right]+\frac{w_C(1+\beta)\Xi}{\Psi w_C\Upsilon+2\sigma(1+\beta)(1-w_C)}\theta_y.$$

Case IIA: $\Upsilon<\frac{\sigma(1-w_C)}{w_C}-\frac{(\varphi+\sigma)}{w_C(1-\lambda)(1+\varphi)}\left[w_C\lambda+\frac{\sigma\mu_C(1+w_C)(1-w_C)}{w_C}\right]$ and one of the following:

(i) $1-\rho_r-\frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y<\theta_\pi<\frac{1}{1-w_C}$ and $\theta_y>\frac{\theta_\pi}{\Xi}\left[\frac{\Psi\Upsilon}{1+\beta}+\frac{2\sigma(1-w_C)}{w_C}\right]-\frac{(1+\rho_r)}{\Xi}\left(\frac{\Psi\Upsilon}{1+\beta}+\frac{2\sigma}{w_C}\right)$.

(ii) $0<\theta_\pi<\min\left\{1-\rho_r-\frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y,\frac{1}{1-w_C}\right\}$, $\theta_y<\frac{\theta_\pi}{\Xi}\left[\frac{\Psi\Upsilon}{1+\beta}+\frac{2\sigma(1-w_C)}{w_C}\right]-\frac{(1+\rho_r)}{\Xi}\left(\frac{\Psi\Upsilon}{1+\beta}+\frac{2\sigma}{w_C}\right)$ and one of the inequalities given by (C.48)–(C.49) is satisfied.

(iii) $\theta_\pi>\max\left\{1-\rho_r-\frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y,\frac{1}{1-w_C}\right\}$, $\theta_y>\frac{\theta_\pi}{\Xi}\left[\frac{\Psi\Upsilon}{1+\beta}+\frac{2\sigma(1-w_C)}{w_C}\right]-\frac{(1+\rho_r)}{\Xi}\left(\frac{\Psi\Upsilon}{1+\beta}+\frac{2\sigma}{w_C}\right)$ and one of the inequalities given by (C.48)–(C.49) is satisfied.

Case IIB: $\frac{\sigma(1-w_C)}{w_C}-\frac{(\varphi+\sigma)}{w_C(1-\lambda)(1+\varphi)}\left[w_C\lambda+\frac{\sigma\mu_C(1+w_C)(1-w_C)}{w_C}\right]<\Upsilon<0$ and one of the following:

(i) $1-\rho_r-\frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y<\theta_\pi<\frac{1}{1-w_C}$, $\rho_r>-\frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y-\frac{w_C}{1-w_C}$ and $\theta_y<\frac{\theta_\pi}{\Xi}\left[\frac{\Psi\Upsilon}{1+\beta}+\frac{2\sigma(1-w_C)}{w_C}\right]-\frac{(1+\rho_r)}{\Xi}\left(\frac{\Psi\Upsilon}{1+\beta}+\frac{2\sigma}{w_C}\right)$.

(ii) $\frac{1}{1-w_C}<\theta_\pi<1-\rho_r-\frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y$, $\rho_r<-\frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y-\frac{w_C}{1-w_C}$ and $\theta_y>\frac{\theta_\pi}{\Xi}\left[\frac{\Psi\Upsilon}{1+\beta}+\frac{2\sigma(1-w_C)}{w_C}\right]-\frac{(1+\rho_r)}{\Xi}\left(\frac{\Psi\Upsilon}{1+\beta}+\frac{2\sigma}{w_C}\right)$.

(iii) $0<\theta_\pi<\min\left\{1-\rho_r-\frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y,\frac{1}{1-w_C}\right\}$ and one of the inequalities given by (C.48)–(C.49) is satisfied.

(iv) $\theta_\pi>\max\left\{1-\rho_r-\frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y,\frac{1}{1-w_C}\right\}$, $\theta_y<\frac{\theta_\pi}{\Xi}\left[\frac{\Psi\Upsilon}{1+\beta}+\frac{2\sigma(1-w_C)}{w_C}\right]-\frac{(1+\rho_r)}{\Xi}\left(\frac{\Psi\Upsilon}{1+\beta}+\frac{2\sigma}{w_C}\right)$

and one of the inequalities given by (C.48)–(C.49) is satisfied.

The above conditions arise from the following derivation. The dynamic system is given by:

$$c_{t+1}^R + \frac{w_C}{\sigma}\pi_{H,t+1} - \frac{w_C}{\sigma} r_t = c_t^R, \tag{C.50}$$

$$\beta \pi_{H,t+1} = \pi_{H,t} - \Psi \Upsilon c_t^R, \tag{C.51}$$

$$r_t - \frac{\sigma(1-w_C)\theta_\pi}{w_C} c_{t+1}^R - \theta_\pi \pi_{H,t+1} = \rho_r r_{t-1} + \left[\Xi\theta_y - \frac{\sigma(1-w_C)\theta_\pi}{w_C}\right] c_t^R. \tag{C.52}$$

This can be expressed as:

$$\mathbf{z}_{t+1} = \mathbf{A_3}\mathbf{z}_t, \quad \mathbf{z}_t = \left[c_t^R \; \pi_{H,t} \; r_{t-1}\right]',$$

$$\mathbf{A_3} \equiv \begin{bmatrix} 1 - \frac{\Psi w_C \Upsilon(\theta_\pi - 1)}{\beta\sigma[1-(1-w_C)\theta_\pi]} + \frac{w_C \Xi \theta_y}{\sigma[1-(1-w_C)\theta_\pi]} & \frac{w_C(\theta_\pi - 1)}{\beta\sigma[1-(1-w_C)\theta_\pi]} & \frac{\rho_r w_C}{\sigma[1-(1-w_C)\theta_\pi]} \\ -\frac{\Psi\Upsilon}{\beta} & \frac{1}{\beta} & 0 \\ -\frac{\Psi w_C \Upsilon \theta_\pi}{\beta[1-(1-w_C)\theta_\pi]} + \frac{\Xi\theta_y}{1-(1-w_C)\theta_\pi} & \frac{w_C\theta_\pi}{\beta[1-(1-w_C)\theta_\pi]} & \frac{\rho_r}{1-(1-w_C)\theta_\pi} \end{bmatrix}.$$

The three eigenvalues of $\mathbf{A_3}$ are solutions to the cubic equation $r^3 + a_2 r^2 + a_1 r + a_0 = 0$, where $a_2 = -1 - \frac{1}{\beta} - \frac{\rho_r}{1-(1-w_C)\theta_\pi} + \frac{\Psi w_C \Upsilon(\theta_\pi - 1)}{\beta\sigma[1-(1-w_C)\theta_\pi]} - \frac{w_C \Xi\theta_y}{\sigma[1-(1-w_C)\theta_\pi]}$, $a_1 = \frac{1}{\beta} + \frac{(1+\beta)\rho_r}{\beta[1-(1-w_C)\theta_\pi]} + \frac{\Psi w_C \Upsilon \rho_r}{\beta\sigma[1-(1-w_C)\theta_\pi]} + \frac{w_C \Xi \theta_y}{\beta\sigma[1-(1-w_C)\theta_\pi]}$, and $a_0 = -\frac{\rho_r}{\beta[1-(1-w_C)\theta_\pi]}$. With one predetermined variable r_{t-1}, determinacy requires that one eigenvalue is inside the unit circle and two eigenvalues are outside the unit circle. By Proposition C.2 of Woodford (2003), this is the case if and only if either of the following two cases is satisfied: Case I: $1 + a_2 + a_1 + a_0 < 0$, $-1 + a_2 - a_1 + a_0 > 0$, Case II: $1 + a_2 + a_1 + a_0 > 0$, $-1 + a_2 - a_1 + a_0 < 0$, and either $|a_2| > 3$ or $a_0^2 - a_0 a_2 + a_1 - 1 > 0$.

For Case I, the two inequalities reduce to:

$$\frac{\Psi\Upsilon(\theta_\pi - 1 + \rho_r) + (1-\beta)\Xi\theta_y}{1-(1-w_C)\theta_\pi} < 0, \tag{C.53}$$

$$-2(1+\beta)\left(1 + \frac{\rho_r}{[1-(1-w_C)\theta_\pi]}\right) + \frac{\Psi w_C \Upsilon(\theta_\pi - 1 - \rho_r) - w_C(1+\beta)\Xi\theta_y}{\sigma\left[1-(1-w_C)\theta_\pi\right]} > 0. \tag{C.54}$$

First assume that $\Upsilon > 0$. If $\Psi\Upsilon(\theta_\pi - 1 + \rho_r) + (1-\beta)\Xi\theta_y > 0$, conditions (C.53) and

(C.54) require:

$$\frac{1}{1-w_C} < \theta_\pi < \frac{(1+\rho_r)\left[\Psi w_C \Upsilon + 2\sigma(1+\beta)\right]}{\Psi w_C \Upsilon + 2\sigma(1+\beta)(1-w_C)} + \frac{w_C(1+\beta)\Xi}{\Psi w_C \Upsilon + 2\sigma(1+\beta)(1-w_C)}\theta_y. \quad \text{(C.55)}$$

For the determinacy region to be non-empty requires $\left(\frac{w_C}{1-w_C}\right)\left[\frac{\Psi w_C \Upsilon - (1-w_C)(1+\beta)\Xi\theta_y}{\Psi w_C \Upsilon + 2\sigma(1+\beta)}\right] < \rho_r$. If $\Psi\Upsilon(\theta_\pi - 1 + \rho_r) + (1-\beta)\Xi\theta_y < 0$, condition (C.54) can never be satisfied since $\Xi > 0$. Now assume that $\Upsilon < 0$. If $\Psi\Upsilon(\theta_\pi - 1 + \rho_r) + (1-\beta)\Xi\theta_y > 0$, condition (C.53) requires:

$$\Xi > 0 \Leftrightarrow \Upsilon > \frac{\sigma(1-w_C)}{w_C} - \frac{(\varphi+\sigma)}{w_C(1-\lambda)(1+\varphi)}\left[w_C\lambda + \frac{\sigma\mu_C(1+w_C)(1-w_C)}{w_C}\right] \quad \text{(C.56)}$$

and

$$\frac{1}{1-w_C} < \theta_\pi < 1 - \rho_r - \frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y. \quad \text{(C.57)}$$

For the determinacy region to be non-empty $\rho_r < -\frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y - \frac{w_C}{1-w_C}$, while condition (C.54) requires:

$$\theta_y > \frac{\theta_\pi}{\Xi}\left[\frac{\Psi\Upsilon}{1+\beta} + \frac{2\sigma(1-w_C)}{w_C}\right] - \frac{(1+\rho_r)}{\Xi}\left(\frac{\Psi\Upsilon}{1+\beta} + \frac{2\sigma}{w_C}\right). \quad \text{(C.58)}$$

If $\Psi\Upsilon(\theta_\pi - 1 + \rho_r) + (1-\beta)\Xi\theta_y < 0$, condition (C.53) requires that $1 - \rho_r - \frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y < \theta_\pi < \frac{1}{1-w_C}$ and (C.54) requires (C.58) if $\Xi < 0$. Otherwise

$$\theta_y < \frac{\theta_\pi}{\Xi}\left[\frac{\Psi\Upsilon}{1+\beta} + \frac{2\sigma(1-w_C)}{w_C}\right] - \frac{(1+\rho_r)}{\Xi}\left(\frac{\Psi\Upsilon}{1+\beta} + \frac{2\sigma}{w_C}\right) \quad \text{(C.59)}$$

and $\rho_r > -\frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y - \frac{w_C}{1-w_C}$ for the determinacy region to be non-empty.

For Case II, the first two inequalities reduce to:

$$\frac{\Psi\Upsilon(\theta_\pi - 1 + \rho_r) + (1-\beta)\Xi\theta_y}{1-(1-w_C)\theta_\pi} > 0, \quad \text{(C.60)}$$

$$-2(1+\beta)\left(1 + \frac{\rho_r}{[1-(1-w_C)\theta_\pi]}\right) + \frac{\Psi w_C \Upsilon(\theta_\pi - 1 - \rho_r) - w_C(1+\beta)\Xi\theta_y}{\sigma\left[1-(1-w_C)\theta_\pi\right]} < 0. \quad \text{(C.61)}$$

First assume that $\Upsilon > 0$. Equation (C.60) requires that $1 - \rho_r - \frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y < \theta_\pi < \frac{1}{1-w_C}$ and (C.61) requires the upper-bound on θ_π given by (C.55). The remaining inequalities give (C.48) and (C.49). Now assume that $\Upsilon < 0$. Condition (C.60) requires either (i) $0 < \theta_\pi < \min\left\{1 - \rho_r - \frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y, \frac{1}{1-w_C}\right\}$ or (ii) $\max\left\{1 - \rho_r - \frac{(1-\beta)\Xi}{\Psi\Upsilon}\theta_y, \frac{1}{1-w_C}\right\} < \theta_\pi$. For (i), (C.61) is always satisfied if $\Xi > 0$. Otherwise, the upper-bound on θ_y given by (C.57) is needed. The remaining inequalities give (C.48) and (C.49). For (ii), condition (C.61) requires (C.59) if $\Xi > 0$ and (C.58) if $\Xi < 0$. The remaining inequalities give (C.48) and (C.49). This completes the proof.

C.7 Determinacy Conditions under a Contemporaneous-Looking Rule

Proposition 8. ***(Current-looking rule)*** *If the interest-rate rule reacts to current-looking CPI inflation with interest-rate inertia, the necessary and sufficient conditions for determinacy are:*

Case I: $\Upsilon > 0,\ \ \theta_\pi > \max\{0, 1 - \rho_r\}$*, and one of the following inequalities is satisfied:*

$$-1 - \frac{1}{\beta} - \rho_r - \theta_\pi(1 - w_C) - \frac{\Psi w_C \Upsilon}{\beta\sigma} < -3, \tag{C.62}$$

$$[\theta_\pi(1 - w_C) + \rho_r]\left[[\theta_\pi(1 - w_C) + \rho_r]\frac{(1-\beta)}{\beta} + \beta - \frac{1}{\beta} - \frac{\Psi w_C \Upsilon}{\beta\sigma}\right] \\ +1 - \beta + \frac{\Psi w_C \Upsilon\,(\theta_\pi + \rho_r)}{\sigma} > 0. \tag{C.63}$$

Case IIA: $\Upsilon < 0,\ \ \theta_\pi > \max\{0, 1 - \rho_r\},$ *and* $\Upsilon < -\frac{2\sigma(1+\beta)[1+\rho_r+\theta_\pi(1-w_C)]}{\Psi w_C(1+\rho_r+\theta_\pi)}$.

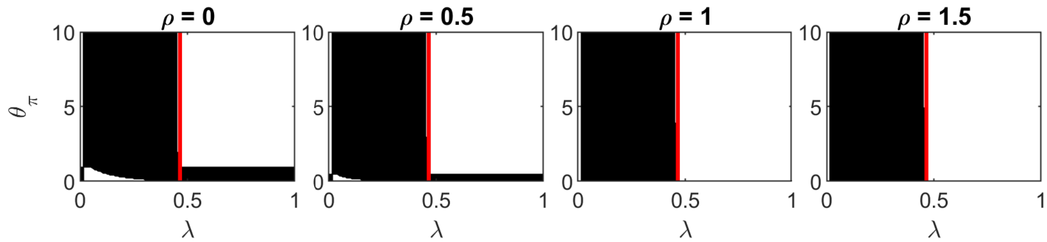
Case IIB: $\Upsilon < 0,\ \ 0 < \theta_\pi < 1 - \rho_r,\ \ \Upsilon > -\frac{2\sigma(1+\beta)[1+\rho_r+\theta_\pi(1-w_C)]}{\Psi w_C(1+\rho_r+\theta_\pi)}$*, and either* (C.63) *or the following inequality*

$$\left|-1 - \frac{1}{\beta} - \rho_r - \theta_\pi(1 - w_C) - \frac{\Psi w_C \Upsilon}{\beta\sigma}\right| < -3 \tag{C.64}$$

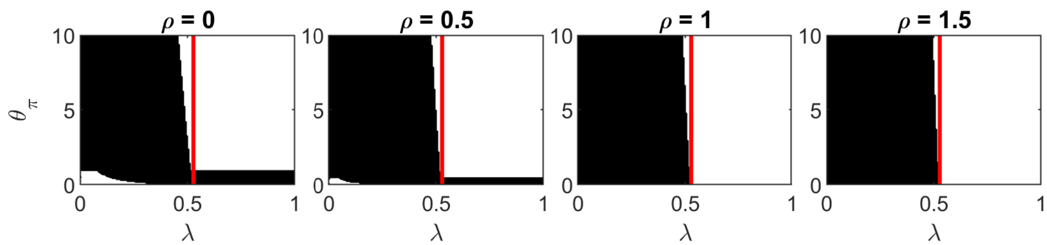
is satisfied.

Proof. The dynamic system is given by:

$$c^R_{t+1} + \frac{w_C}{\sigma}\pi_{H,t+1} - \frac{w_C}{\sigma}r_t = c^R_t, \tag{C.65}$$



(a) Open economy



(b) Closed economy

Figure 12: Determinacy regions for small model with current-CPI inflation feedback rule. Parameter values are $\Psi = 0.086$, $\varphi = \sigma = 2$, $\zeta = 7$, $\beta = 0.99$, and $w_C = 0.6$, $\mu_C = 0.62$ for the open economy (top panel) and $w_C = 1$ for the closed economy (bottom panel). The red vertical line gives λ^* below which IADL holds.

$$\beta \pi_{H,t+1} = \pi_{H,t} - \Psi \Upsilon c_t^R, \tag{C.66}$$

$$r_t = \rho_r r_{t-1} + \theta_\pi \pi_t = \left[\rho_r + \theta_\pi (1 - w_C)\right] r_{t-1} + w_C \theta_\pi \pi_{H,t}. \tag{C.67}$$

This can be expressed as:

$$\mathbf{z}_{t+1} = \mathbf{A_4} \mathbf{z}_t, \quad \mathbf{z}_t = \left[c_t^R \; \pi_{H,t} \; r_{t-1}\right]',$$

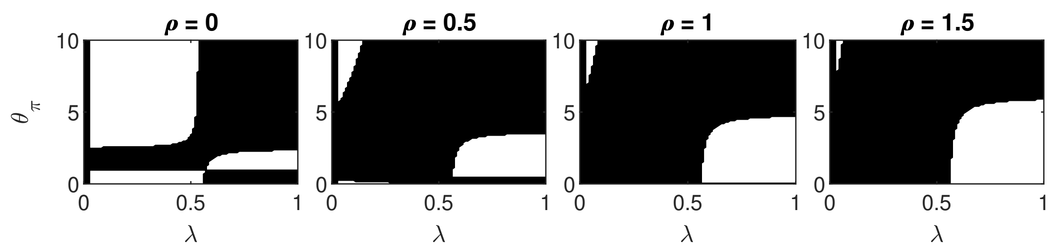
$$\mathbf{A_4} \equiv \begin{bmatrix} 1 + \frac{\Psi w_C \Upsilon}{\beta\sigma} & \frac{w_C}{\sigma}\left(w_C \theta_\pi - \frac{1}{\beta}\right) & \frac{w_C}{\sigma}\left[\rho_r + \theta_\pi(1 - w_C)\right] \\ -\frac{\Psi\Upsilon}{\beta} & \frac{1}{\beta} & 0 \\ 0 & w_C \theta_\pi & \rho_r + \theta_\pi (1 - w_C) \end{bmatrix}.$$

The three eigenvalues of $\mathbf{A_4}$ are solutions to the cubic equation $r^3 + a_2 r^2 + a_1 r + a_0 = 0$, where $a_2 = -1 - \frac{1}{\beta} - \rho_r - \theta_\pi(1 - w_C) - \frac{\Psi w_C \Upsilon}{\beta\sigma}$, $a_1 = \frac{1}{\beta} + \frac{\theta_\pi(1-w_C)(1+\beta)}{\beta} + \frac{\Psi w_C \Upsilon \theta_\pi}{\beta\sigma} + \rho_r\left(1 + \frac{1}{\beta} + \frac{\Psi w_C \Upsilon}{\beta\sigma}\right)$, and $a_0 = -\frac{\theta_\pi(1-w_C)}{\beta} - \frac{\rho_r}{\beta}$. With one predetermined variable r_t, determinacy requires that one eigenvalue is inside the unit circle and two eigenvalues are outside the unit circle. By Proposition C.2 of Woodford (2003), this is the case if and only if either of the following two cases is satisfied: Case 1: $1+a_2+a_1+a_0 < 0$, $-1+a_2-a_1+a_0 > 0$, Case 2: $1+a_2+a_1+a_0 > 0$, $-1+a_2-a_1+a_0 < 0$, and either $|a_2| > 3$ or $a_0^2 - a_0 a_2 + a_1 - 1 > 0$. For Case I, the second inequality can never be satisfied with $\Upsilon > 0$. With $\Upsilon < 0$, the first inequality of Case I requires $\theta_\pi > 1 - \rho_r$ and the second inequality yields reduces to $\Upsilon > -\frac{2\sigma(1+\beta)[1+\rho_r+\theta_\pi(1-w_C)]}{\Psi w_C(1+\rho_r+\theta_\pi)}$. For Case II, the two inequalities are satisfied if $\theta_\pi > 1 - \rho_r$, provided $\Upsilon > 0$. The remaining inequalities give (C.62) and (C.63). If $\Upsilon < 0$, the first inequality requires $0 < \theta_\pi < 1 - \rho_r$ and the second inequality $\Upsilon > -\frac{2\sigma(1+\beta)[1+\rho_r+\theta_\pi(1-w_C)]}{\Psi w_C(1+\rho_r+\theta_\pi)}$. The remaining inequalities give (C.63) and (C.64). This completes the proof. □

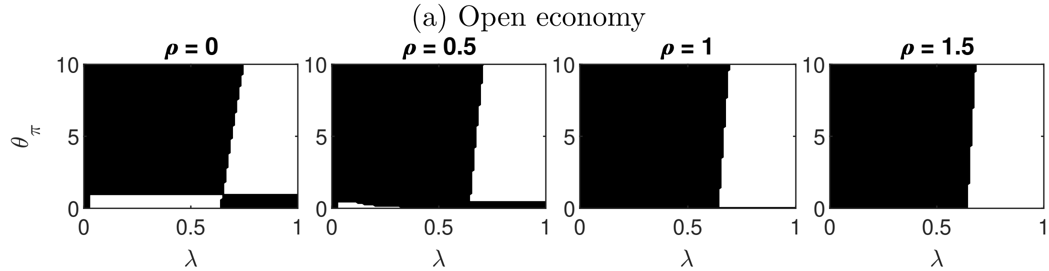
The analytical conditions generate similar conclusions to a forward-looking domestic price inflation rule. As shown in Figure 12, policy inertia shrinks the determinate policy space, whereas trade openness enlarges the determinate policy space under SADL and shrinks it under IADL.

C.8 Determinacy Analysis under Incomplete Asset Markets, Trend Inflation and Capital

Figures 13–15 show determinacy regions under incomplete asset markets (no capital, zero trend inflation); incomplete asset markets, capital in production and 4% trend inflation; and incomplete asset markets, capital in production and 6% trend inflation respectively.

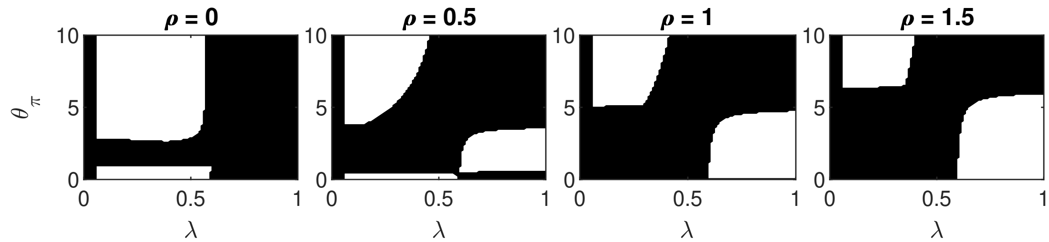


(a) Open economy

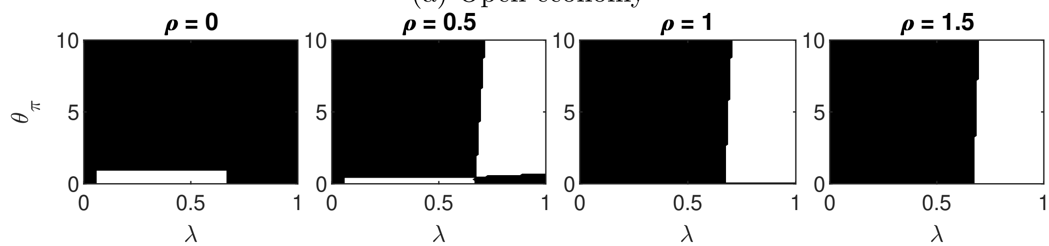


(b) Closed economy

Figure 13: Determinacy regions (white areas) for the LAMP model with incomplete asset markets. Parameterization is given in Table 2 of appendix B.7. $w_C = 0.6$ for the open economy (top panel) and $w_C = 1$ for the closed economy (bottom panel).

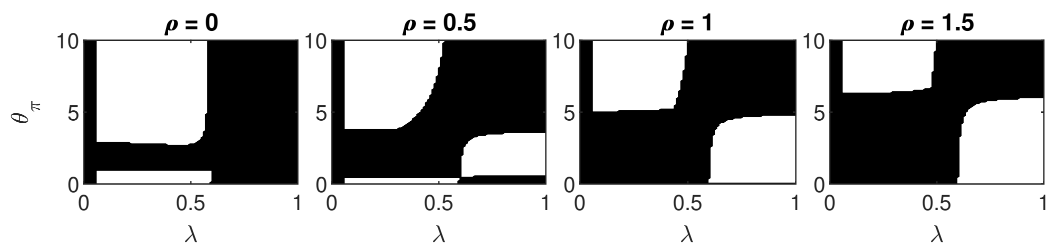


(a) Open economy

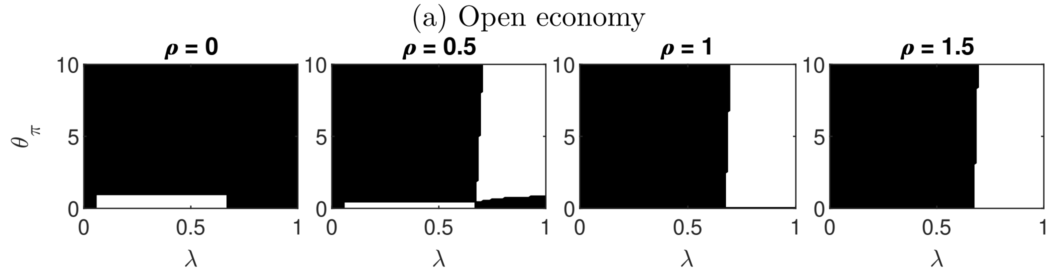


(b) Closed economy

Figure 14: Determinacy regions (white areas) for the LAMP model with incomplete asset markets, capital and 4% trend inflation. Parameterization is given in Table 2 of appendix B.7. $w_C = w_I = 0.6$ for the open economy (top panel) and $w_C = w_I = 1$ for the closed economy (bottom panel).



(a) Open economy

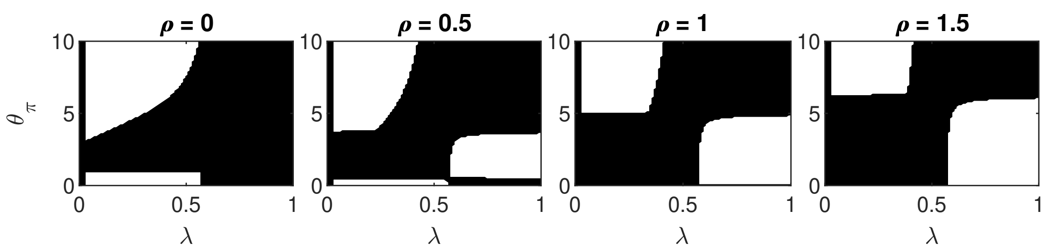


(b) Closed economy

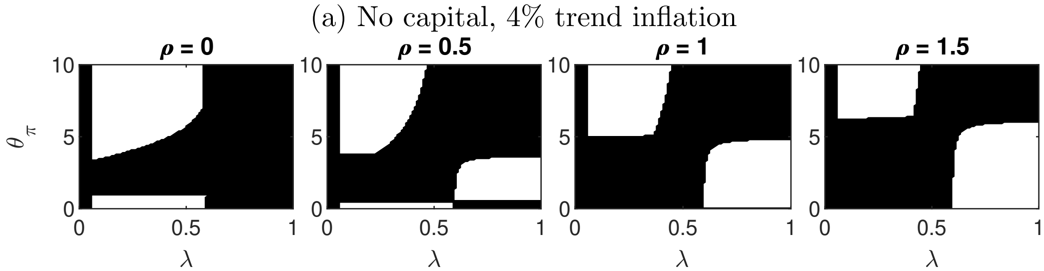
Figure 15: Determinacy regions (white areas) for the LAMP model with incomplete asset markets, capital and 6% trend inflation. Parameterization is given in Table 2 of appendix B.7. $w_C = w_I = 0.6$ for the open economy (top panel) and $w_C = w_I = 1$ for the closed economy (bottom panel).

C.9 Determinacy Analysis under Dominant Currency Pricing

Figure 16 shows determinacy regions under dominant (or local) currency pricing and incomplete asset markets. The top panel is with 4% trend inflation but without capital, and the bottom shows both capital in production and 4% trend inflation.



(a) No capital, 4% trend inflation



(b) Capital in production, 4% trend inflation

Figure 16: Determinacy regions (white areas) for the LAMP model with incomplete asset markets and dominant currency pricing. Parameterization is given in Table 2 of appendix B.7.

C.10 Determinacy Analysis at the ZLB

This subsection overviews the environment and tests used to study the determinacy properties of the model with a zero lower bound (ZLB) on the nominal interest rate. The necessary and sufficient conditions are discussed in detail in Holden (2022). First, note that the interest-rate rule with a ZLB can be written as:

$$r_t = \rho_r r_{t-1} + \theta_\pi \pi_{t+1} + \eta_t, \tag{C.68}$$

where η_t is a partially anticipated add-factor defined as:

$$\eta_t \equiv \max\{0, \bar{r} + \rho_r r_{t-1} + \theta_\pi \pi_{t+1}\} - \bar{r} + \rho_r r_{t-1} + \theta_\pi \pi_{t+1}. \tag{C.69}$$

Because η_t is partially predictable it can be considered as a monetary policy news shock; information that the ZLB will bind in k periods ahead is equivalent to news that $\eta_{t+k} > 0$.

Starting with a path for r_t, ignoring the ZLB up to horizon T, the problem of computing the sequence of η_t to impose the ZLB can be characterized as a linear complimentarity problem (LCP). This is convenient because it is a well-studied problem in the mathematics literature and so we can use existing tests to check the uniqueness and determinacy properties of a particular interest-rate rule (see Holden, 2022). Let vector $q \equiv [q_1, \cdots, q_T]'$ be the path of $r_t + \bar{r}$ ignoring the bound up to horizon T, and let M be a $T \times T$ matrix where the nth column gives the values of $[r_1, \cdots, r_T]$ conditional on an anticipated news shock, η_n of size 1 in period $t = n$. Given the otherwise linearity of the model, conditional on a path ignoring the bound q, and sequence of news shocks $\eta \equiv [\eta_1, \cdots, \eta_T]'$, the path of the interest rate is given by:

$$r + \bar{r} = q + M\eta, \tag{C.70}$$

where $r \equiv [r_1, \cdots, r_T]'$. M and q are readily solved using the linear model without a ZLB. The LCP(q, M) is to solve the vector η to satisfy the following constraints:

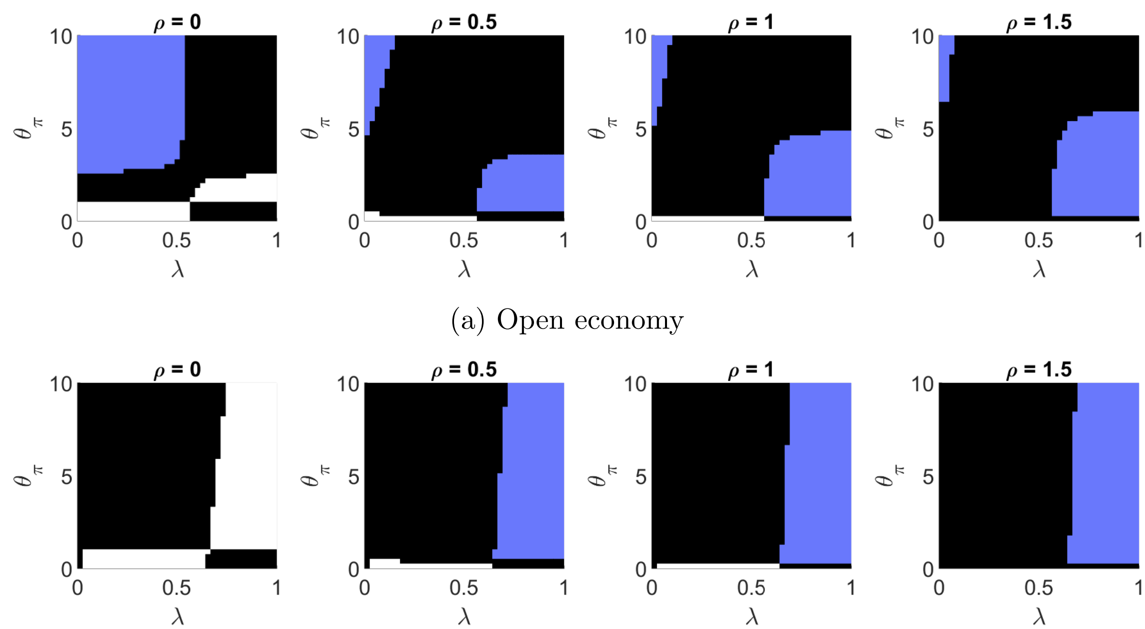
$$\eta \geq 0, \tag{C.71}$$

$$q + M\eta \geq 0, \tag{C.72}$$

$$y'(q + M\eta) = 0. \tag{C.73}$$

The above conditions are that news shocks must always be positive (C.71), the ZLB must not be violated (C.72), and the complimentary slackness condition (C.73) which requires that news shocks are only non-zero when the ZLB is binding. To determine whether there are multiple equilibria or explosiveness (infeasibility) requires checking the properties of matrix M.

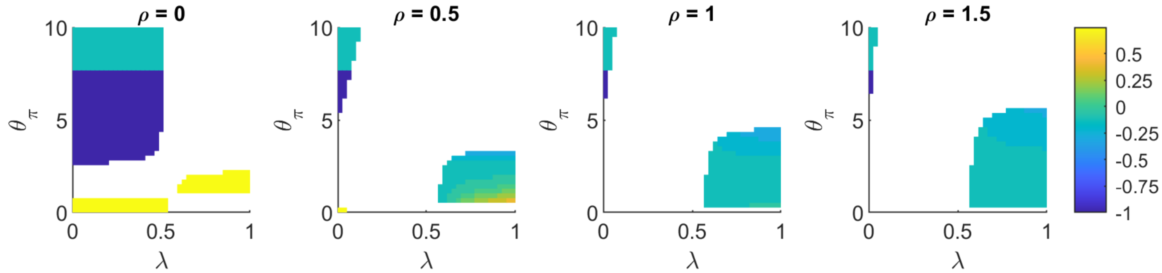
Cottle, Pang and Stone (2009, ch. 3) show that uniqueness is guaranteed if, for all $z \in \mathbb{R}^{T\times 1}$ with $z \neq 0$, there exists $t \in \{1, ..., T\}$ such that $z_t\,(Mz)_t > 0$. Following the notation of Cottle et al. (2009) and Holden (2022), we refer to a matrix satisfying this condition as a **P**-matrix. This is a particular definition of positivity and to gain some intuition, if M is a **P**-matrix in our model, then monetary policy shocks must increase nominal interest rates. This is consistent with the above description of a self-fulfilling ZLB episode which relies on news shocks lowering nominal rates. A full test to determine whether M is a **P**-matrix



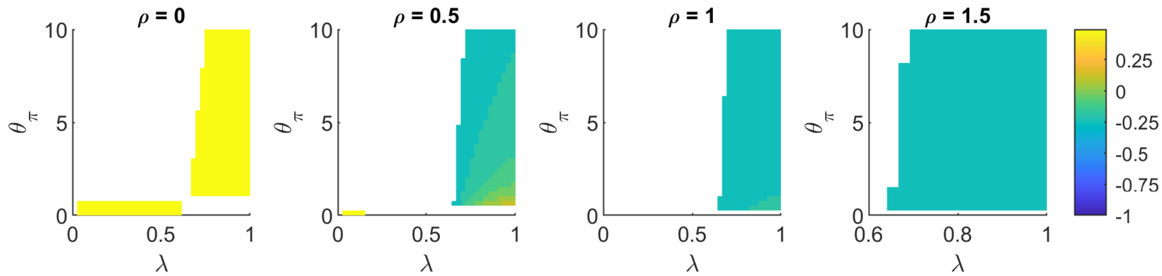
(a) Open economy

(b) Closed economy

Figure 17: Initial tests for multiplicity. The black area represents indeterminacy in the baseline linear LAMP model, the white area indicates there is always a unique equilibrium conditional on households expecting to be away from the ZLB in 200 quarters. Multiplicity cannot be ruled out for the blue area. Parameterization is the same as in Figure 2 of the main text.



(a) Open economy



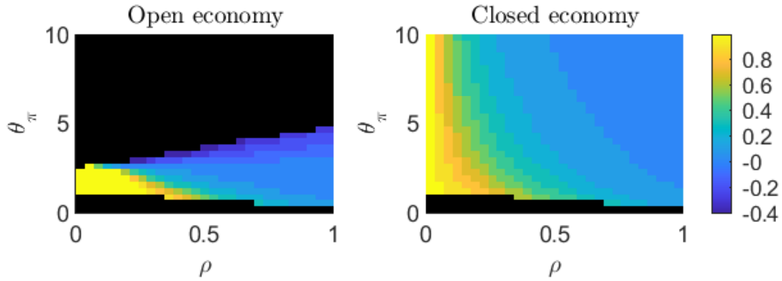
(b) Closed economy

Figure 18: Minimum determinant of the 5×5 leading principal sub-matrix of M. A positive values implies M is a **P**-matrix.

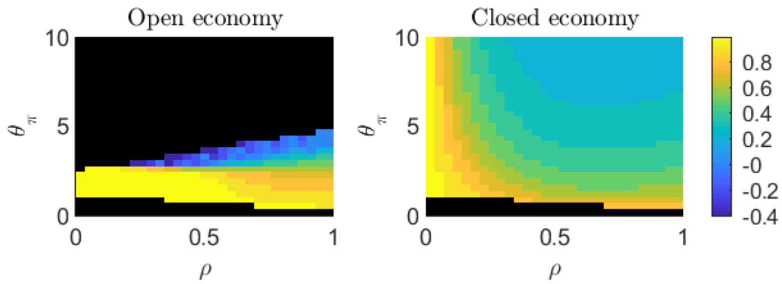
may be infeasible for a large horizon T, however it is possible to check other necessary and sufficient conditions. For example, M is definitely a **P**-matrix if it is symmetric positive definite and it is definitely not a **P**-matrix if it has any complex eigenvalues outside the interval $\left(-\pi + \frac{\pi}{T}\,,\, \pi - \frac{\pi}{T},\right)$ (see Holden, 2022).[39] Figure 17 shows the results of the initial checks where, as before, the black area represents calibrations leading to indeterminacy in the baseline linear LAMP model. The white area now represents calibrations for which uniqueness is guaranteed providing the economy is expected to be away from the ZLB in 200 quarters. These initial checks show that when a determinate policy rule is available under IADL, uniqueness is always guaranteed except for high values of $\theta_\pi > \max\left\{\frac{1}{1-w_c}, \Gamma_1\right\}$. Under SADL, we cannot rule out multiplicity except when interest rate inertia is absent.

As an alternative, we can look to other indicative statistics such as the minimum determinant of a principal sub-matrix of M. When this is positive, M is a **P**-matrix. This is useful as it is a continuous measure and so allows us to gain insight as to whether a pa-

[39] Refer to Corollaries 4 and 5 in appendix C of Holden (2022) for the necessary and sufficient conditions.



(a) Lagged interest rate in policy rule



(b) Lagged shadow rate in policy rule

Figure 19: The role of trade openness and policy inertia for the baseline model under a ZLB setting $\lambda = 1$. The black areas represent indeterminacy in the linear model, the high values in yellow correspond to better determinacy properties and low values in blue to worse properties. Values >1 truncated to 1, and values < -0.4 to 0.4.

rameter worsens or improves the multiplicity properties of the model. Figure 18 presents the results of this. A visual check of this reveals that policy inertia worsens the problem of multiple equilibria under SADL. Increasing the response of policy to inflation also worsens this problem. The reason being that a more aggressive policy stance will cut the interest rate further if a future contraction is expected. It is this mechanism that can lead to a self-fulfilling ZLB episode.

C.10.1 The Role of Trade Openness and Policy Inertia on the stability properties at the ZLB

We can look further at how policy inertia and trade openness affect the determinacy properties of the model under a ZLB using other indicative statistics. Figure 19 shows the minimum determinant of a principal sub-matrix of M, where M is a 5×5 matrix containing impulse response functions to a positive monetary policy news shocks at different horizons up to $T = 5$. When this determinant is positive, uniqueness is guaranteed (up to $T = 5$).[40]

We focus on the SADL case and set $\lambda = 1$.[41] For the interest-rate rule (3.15), except for small values of ρ_r, higher policy inertia worsens the determinacy properties of both the closed and open economy versions of the model. However, by including the lagged shadow rate (3.16), policy inertia tends to improve the determinacy conditions, except for a small interval of θ_π in the open economy.

D Optimal Policy: Derivations and Proofs

D.1 Proof of Proposition 6

Under LAMP, the SOE government chooses monetary policy to maximize a utilitarian social welfare given by $\lambda U(C_t^R, N_t^R) + (1-\lambda)U(C_t^C, N_t^C)$, where from (2.1) as $\sigma \to 1$

$$U(C_t^i, N_t^i) = \log C_t^i - \frac{\left(N_t^i\right)^{1+\varphi}}{1+\varphi}, \quad i = R, C. \tag{D.1}$$

To approximate this welfare criterion we implement the standard algorithm of Taylor-series expansion, in particular following the steps in Woodford (2003*a*), Benigno and Woodford (2004), Galí and Monacelli (2005), Bilbiie (2008), and Levine, Pearlman and Pierse (2008).

D.1.1 Step 1: Taylor Series Expansion

Taking a second-order Taylor linear expansion we get:

$$\begin{aligned} U\left(C_t^i, N_t^i\right) &\approx U\left(\overline{C}_t^i, \overline{N}_t^i\right) + U_{C^i}\left(C_t^i - \overline{C}_t^i\right) + U_{N^i}\left(N_t^i - \overline{N}_t^i\right) \\ &+ \frac{1}{2}\left[U_{C^iC^i}\left(C_t^i - \overline{C}_t^i\right)^2 + 2\,U_{C^iN^i}\left(C_t^i - \overline{C}_t^i\right)\left(N_t^i - \overline{N}_t^i\right) + U_{N^iN^i}\left(N_t^i - \overline{N}_t^i\right)^2\right] \\ i &= R, C \qquad \text{up to second order terms.} \end{aligned} \tag{D.2}$$

[40] To understand why, consider that the determinant of M can be thought of as equivalent to a measure of volume. The sign of the determinant gives information on the positivity of the response of monetary policy to news shocks at different horizons. Recall from the earlier intuition of a ZLB-induced sunspot shock that a self-fulfilling ZLB episode relies on a negative response of monetary policy to a positive monetary policy shock at some horizon.

[41] Trade openness and policy inertia do not affect the outcomes under IADL, since we always have uniqueness under IADL unless we are in the blue regions highlighted in Figure 6, when we always have multiplicity.

Defining $c_t^i \equiv \frac{C_t^i - \overline{C}_t^i}{\overline{C}_t^i}$ and $n_t^i \equiv \frac{N_t^i - \overline{N}_t^i}{\overline{N}_t^i}$ to be relative deviations about $\overline{C}_t^i$ or $\overline{N}_t^i$, which can be steady states or flexi-price equilibria, (D.2) becomes:

$$\begin{aligned} U\left(C_t^i, N_t^i\right) &\approx U\left(\overline{C}_t^i, \overline{N}_t^i\right) + U_{C^i}\overline{C}_t^i c_t^i + U_{N_t^i}\overline{N}_t^i n_t^i \\ &+ \frac{1}{2}\left[U_{C^iC^i}(\overline{C}_t^i)^2(c_t^i)^2 + 2\,U_{C^iN^i}\overline{C}^i\overline{N}^i c_t^i n_t^i + U_{N^iN^i}(\overline{N}^i)^2(n_t^i)^2\right],\ i = R, C. \end{aligned} \tag{D.3}$$

(D.3) is completely general. Adopting our particular choice of preferences (D.1), we have that $U_{C^iN^i} = 0$, $U_{C^i} = (\overline{C}_t^i)^{-1}$, $U_{C^iC^i} = -(\overline{C}_t^i)^{-2}$, $U_{N^i} = -(\overline{N}_t^i)^{\varphi}$, $U_{N^iN^i} = -\varphi(\overline{N}_t^i)^{\varphi-1}$. Then (D.3) becomes:

$$\begin{aligned} U\left(C_t^i, N_t^i\right) &\approx U\left(\overline{C}_t^i, \overline{N}_t^i\right) + c_t^i - (\overline{N}_t^i)^{1+\varphi} n_t^i \\ &- \frac{1}{2}\left[(c_t^i)^2 + \varphi(\overline{N_t}^i)^{1+\varphi}(n_t^i)^2\right],\ i = R, C. \end{aligned}$$

Hence, the social welfare criterion, wel_t, is given approximately up to second order terms by:

$$\begin{aligned} wel_t &= U(C_t^R, N_t^R) + (1-\lambda)U(C_t^C, N_t^C) \approx U(\overline{C}_t^R, \overline{N}_t^R) + (1-\lambda)U(\overline{C}_t^C, \overline{N}_t^C) \\ &+ \lambda c_t^R + (1-\lambda)c_t^C - \lambda(\overline{N}_t^R)^{1+\varphi} n_t^R + (1-\lambda)(\overline{N}_t^C)^{1+\varphi} n_t^C \\ &- \frac{1}{2}\left[\lambda\left((c_t^R)^2 + \varphi(\overline{N_t}^R)^{1+\varphi}(n_t^R)^2\right) + (1-\lambda)\left((c_t^C)^2 + \varphi(\overline{N_t}^C)^{1+\varphi}(n_t^C)^2\right)\right]. \end{aligned} \tag{D.4}$$

(D.4) holds for our particular choice of household preferences for any baseline $U(\overline{C}_t^i, \overline{N}_t^i)$, about which the Taylor-series expansion (or approximation) is based. In our paper this is the distorted equitable steady state. We now choose the optimal equitable flexi-price equilibrium with a welfare-relevant output gap $x_{1,t}$ for which $(\overline{N}_t^R)^{1+\varphi} = (\overline{N}_t^C)^{1+\varphi} = w_C$. Then (D.4) becomes:

$$\begin{aligned} wel_t &= \lambda U(C_t^R, N_t^R) + (1-\lambda)U(C_t^C, N_t^C) \approx \lambda U(\overline{C}_t^R, \overline{N}_t^R) + (1-\lambda)U(\overline{C}_t^C, \overline{N}_t^C) \\ &+ \lambda c_t^R + (1-\lambda)c_t^C - w_C\left[\lambda n_t^R + (1-\lambda)n_t^C\right] \\ &- \frac{1}{2}\left[\lambda\left((c_t^R)^2 + \varphi w_C(n_t^R)^2\right) + (1-\lambda)\left((c_t^C)^2 + \varphi w_C(n_t^C)^2\right)\right]. \end{aligned} \tag{D.5}$$

D.1.2 Step 2: Use of the Resource Constraint in Linearized Form

To express this as a quadratic form we now impose the resource constraint which can be expressed for our purposes as:

$$C_t^{w_C}(C_t^R)^{1-w_C} = Y_t^{w_C}((C_t^{R*})^{1-w_C})^{1-w_C}, \tag{D.6}$$

$$Y_t = \frac{A_t N_t}{\Delta_t}, \tag{D.7}$$

$$N_t = \lambda N_t^R + (1-\lambda)N_t^C, \tag{D.8}$$

$$C_t = \lambda C_t^R + (1-\lambda)C_t^C. \tag{D.9}$$

Denoting any variable Z_t in log-deviation form $\hat{z}_t \equiv \log\left(Z_t/\bar{Z}_t\right)$ and in relative deviation form by $z_t = (Z_t - \bar{Z}_t)/\bar{Z}_t$, a Taylor-series expansion gives

$$z_t \approx \widehat{z}_t + \frac{1}{2}\widehat{z}_t^2 \text{ (up to second order).} \tag{D.10}$$

In what follows, we take $\bar{Z}_t$ to be the equitable flexi-price equilibrium supported by tax subsidies set out in Proposition 3. Then $x_t = (Y_t - \bar{Y}_t)/\bar{Y}_t$ becomes the output gap.

Taking logs, (D.6) and (D.7) can be written exactly as:

$$w_C\widehat{c}_t + (1-w_C)\widehat{c}_t^R = w_C\widehat{x}_t + \text{ t.i.p.}, \tag{D.11}$$

$$\widehat{x}_t = \widehat{n}_t - \widehat{\delta}_t + \text{ t.i.p.}, \tag{D.12}$$

where terms independent of policy (t.i.p.) are those only involving shock processes y_t^* and a_t.

Now consider the linear term in (D.5) which can be written simply as $c_t - w_C n_t = c_t - w_C(x_t + \delta_t)$ plus t.i.p.. Using (D.10) we have up to o(2):

$$\begin{aligned} c_t - w_C n_t &= \lambda c_t^R + (1-\lambda)c_t^C - w_C\left[\lambda n_t^R + (1-\lambda)n_t^C\right] \\ &\approx \lambda\widehat{c}_t^R + (1-\lambda)\widehat{c}_t^C - w_C\left[\lambda(\widehat{n}_t^R + (1-\lambda)\widehat{n}_t^C\right] \\ &+ \frac{1}{2}\left[\lambda(\widehat{c}_t^R)^2 + (1-\lambda)(\widehat{c}_t^C)^2 - w_C\left[\lambda((\widehat{n}_t^R)^2 + (1-\lambda)(\widehat{n}_t^C)^2\right]\right]. \end{aligned}$$

Then (D.5) becomes:

$$wel_t = wel + \widehat{c}_t - w_C \widehat{n}_t - \frac{1}{2}\left[\lambda\left((1+\varphi)w_C(n_t^R)^2\right) + (1-\lambda)\left((1+\varphi)w_C(n_t^C)^2\right)\right]. \quad \text{(D.13)}$$

Using the exact log-linear resource constraint (D.11) we then have:

$$\begin{aligned}\widehat{c}_t - w_C\widehat{x}_t &= (1-w_C)\left(\widehat{x}_t - \frac{1}{w_C}\widehat{c}_t^R\right) \\ &= \frac{1-w_C}{w_C}(w_C\widehat{x}_t - \widehat{c}_t^R) \\ &= \frac{1-w_C}{w_C}(w_C\widehat{x}_t - \widehat{c}_t + \widehat{c}_t - \widehat{c}_t^R). \quad \text{(D.14)}\end{aligned}$$

Hence, solving for $\widehat{c}_t - w_C\widehat{x}_t$ we arrive at

$$\widehat{c}_t - w_C\widehat{x}_t = (1-w_C)(\widehat{c}_t - \widehat{c}_t^R). \quad \text{(D.15)}$$

To complete the transformation of (D.15) into second-order terms we recall relevant results for the linearization of our model in log-deviation form from Appendix C.1:

$$\begin{aligned}\widehat{w}_t &= \varphi\widehat{n}_t^R + \sigma\widehat{c}_t^R, \\ \widehat{w}_t &= \varphi\widehat{n}_t^C + \sigma\widehat{c}_t^C, \\ n_t^C &= \frac{\varphi(1-\sigma)}{\varphi+\sigma}n_t^R + \frac{\sigma(1-\sigma)}{\varphi+\sigma}c_t^R, \\ \widehat{x}_t &= \widehat{n}_t - \widehat{\delta}_t + \text{t.i.p.}.\end{aligned}$$

Setting $\sigma = 1$ we get:

$$\begin{aligned}n_t^C &= \widehat{n}_t^C = 0, \\ \widehat{c}_t^C - \widehat{c}_t^R &= -\varphi(\widehat{n}_t^C - \widehat{n}_t^R) = \varphi\widehat{n}_t^R, \\ \widehat{x}_t &= \lambda\widehat{n}_t^R - \widehat{\delta}_t + \text{t.i.p.}.\end{aligned}$$

Using these results we obtain:

$$\widehat{c}_t - w_C\widehat{x}_t = (1-w_C)(\widehat{c}_t - \widehat{c}_t^R)$$

$$
\begin{aligned}
&= (1-w_C)(c_t - c_t^R + o(2)) = (1-w_C)(1-\lambda)(c_t^C - c_t^R + o(2)) \\
&= (1-w_C)(1-\lambda)(\varphi n_t^R + o(2)) = \frac{(1-w_C)(1-\lambda)}{\lambda}\varphi(x_t + o(2)).
\end{aligned}
$$

In what follows, we consider the case where $\frac{(1-w_C)(1-\lambda)\varphi}{\lambda}$ is small[42] and of the same order as deviations of variables about the baseline flexi-price equilibrium allocation or steady state. For example, even for a small open economy the share of imported consumption goods $(1-w_C)$ is typically less than 0.3. Further, if the share of RoT consumers $(1-\lambda) < 0.2$ and $\varphi = 2$, then $(1-w_C)(1-\lambda)\varphi/\lambda < 0.15$. For economies with these features we can then treat $(1-w_C)(1-\lambda)\varphi/\lambda x_t$ as o(2).[43]

Gathering our results together we can now write (D.13) up to o(2) as:

$$
wel_t - wel = \frac{(1-w_C)(1-\lambda)\varphi}{\lambda}x_t - w_C\widehat{\delta}_t - \frac{1}{2}\frac{w_C(1+\varphi)x_t^2}{\lambda}. \tag{D.16}
$$

Note that with our distorted equitable steady state, standard derivations lead to an additional linear term $\frac{\Phi\Psi}{\zeta}x_t$ as in the closed economy (see, for example, Galí (2015)).

D.1.3 Step 3: Quadratic Approximation of Dispersion Term

The remaining step is to obtain a quadratic approximation for the price dispersion term $\widehat{\delta}_t$ in (D.16). To do this we use the following results:

$$
\Delta_t = \xi\Pi_{H,t}^{\zeta}\Delta_{t-1} + (1-\xi)\left(\frac{J_t}{JJ_t}\right)^{-\zeta}, \tag{D.17}
$$

$$
\left(\frac{J_t}{JJ_t}\right)^{1-\zeta} = \frac{1-\xi\Pi_{H,t}^{\zeta-1}}{1-\xi}, \tag{D.18}
$$

where $\frac{J_t}{JJ_t} = \frac{P_{H,t}^0}{P_{H,t}}$ is the optimal reset price. This results in $\Delta_t = \Delta(\Pi_{H,t})$.

We now use a second-order Taylor-series expansion about a zero net inflation $\Pi = \Pi_H = 1$ to show that

$$
\delta_t = \xi\delta_{t-1} + \frac{\xi\zeta}{2(1-\xi)}\pi_{H,t}^2. \tag{D.19}
$$

[42]Notice that this term vanishes in the closed TANK economy (of Bilbiie (2008)), with $w_C = 1$, as well as in the SOE model without LAMP (of Galí and Monacelli (2005)), with $\lambda = 1$.

[43]This is analogous to the way small distortions in the steady state are incorporated into a quadratic approximation in the literature.

Proof

First write (D.17) and (D.18) as:

$$\begin{aligned}
\Delta_t &= \xi \Pi_{H,t}^{\zeta} \Delta_{t-1} + (1-\xi)\left(\frac{1-\xi\Pi_{H,t}^{\zeta-1}}{1-\xi}\right)^{-\frac{\zeta}{1-\zeta}} \\
&= \xi \Pi_{H,t}^{\zeta} \Delta_{t-1} + (1-\xi)^{\frac{1}{1-\zeta}}\left(1-\xi\Pi_{H,t}^{\zeta-1}\right)^{\frac{\zeta}{\zeta-1}} \\
&= \xi F(\Pi_{H,t}, \Delta_{t-1}) + (1-\xi)^{\frac{1}{1-\zeta}} G(\Pi_{H,t}). \qquad \text{(D.20)}
\end{aligned}$$

Next we expand $F(\Pi_{H,t}, \Delta_{t-1})$ and $G(\Pi_{H,t})$ as Taylor series up to second order:

$$\begin{aligned}
F(\Pi_{H,t}, \Delta_{t-1}) &= F(\Pi,\Delta) + F_{\Pi}(\Pi,\Delta)(\Pi_{H,t}-\Pi) + F_{\Delta}(\Pi,\Delta)(\Delta_{t-1}-\Delta) \\
&\quad + \frac{1}{2}\left(\begin{array}{c} F_{\Pi\,\Pi}(\Pi,\Delta)(\Pi_{H,t}-\Pi)^2 + 2F_{\Pi\,\Delta}(\Pi,\Delta)(\Pi_{H,t}-\Pi)(\Delta_{t-1}-\Delta) \\ +F_{\Delta\,\Delta}(\Pi,\Delta))(\Delta_{t-1}-\Delta)^2 \end{array}\right) \\
&\quad + \cdots \\
G(\Pi_{H,t}) &= G(\Pi) + G'(\Pi)(\Pi_{H,t}-\Pi) + \frac{1}{2}G''(\Pi)(\Pi_{H,t}-\Pi)^2 + \cdots
\end{aligned}$$

Subtract $\Delta = \xi F(\Pi,\Delta) + (1-\xi)^{\frac{1}{1-\zeta}} G(\Pi)$ from both sides of (D.20) to give:

$$\begin{aligned}
\Delta_t - \Delta &= \xi\Big(F_{\Pi}(\Pi,\Delta)(\Pi_{H,t}-\Pi) + F_{\Delta}(\Pi,\Delta)(\Delta_{t-1}-\Delta) \\
&+ \frac{1}{2}\left(F_{\Pi\,\Pi}(\Pi,\Delta)(\Pi_{H,t}-\Pi)^2 + 2F_{\Pi\,\Delta}(\Pi,\Delta)(\Pi_{H,t}-\Pi)(\Delta_{t-1}-\Delta) + F_{\Delta\,\Delta}(\Pi,\Delta))(\Delta_{t-1}-\Delta)^2\right)\Big) \\
&+ (1-\xi)^{\frac{1}{1-\zeta}}\left(G'(\Pi)(\Pi_{H,t}-\Pi) + \frac{1}{2}G''(\Pi)(\Pi_{H,t}-\Pi)^2\right).
\end{aligned}$$

Hence:

$$\begin{aligned}
\delta_t \equiv \frac{\Delta_t - \Delta}{\Delta} &= \xi\Big(F_{\Pi}(\Pi,\Delta)\Pi\,\pi_{H,t} + F_{\Delta}(\Pi,\Delta)\Delta\,\delta_{t-1} \\
&+ \frac{1}{2}\left(F_{\Pi\,\Pi}(\Pi,\Delta)\Pi^2\,\pi_{H,t}^2 + 2F_{\Pi\,\Delta}(\Pi,\Delta)\Pi\Delta\,\pi_{H,t}\delta_{t-1}) + F_{\Delta\,\Delta}(\Pi,\Delta))\Delta^2\,\delta_{t-1}^2\right)\Big) \\
&+ (1-\xi)^{\frac{1}{1-\zeta}}\left(G'(\Pi)\Pi\pi_{H,t} + \frac{1}{2}G''^2\pi_{H,t}^2\right), \qquad \text{(D.21)}
\end{aligned}$$

up to second-order terms.

From the definitions

$$
\begin{aligned}
F(\Pi, \Delta) &\equiv \Pi^{\zeta}\Delta, \\
G(\Pi) &\equiv \left(1 - \xi\Pi^{\zeta-1}\right)^{\frac{\zeta}{\zeta-1}},
\end{aligned}
$$

we have:

$$
\begin{aligned}
F_{\Pi}(\Pi, \Delta) &= \zeta\Pi^{\zeta-1}\Delta, \\
F_{\Delta}(\Pi, \Delta) &= \Pi^{\zeta}, \\
F_{\Pi\Pi}(\Pi, \Delta) &= \zeta(\zeta-1)\Pi^{\zeta-2}\Delta \\
& G'\zeta^{-2}\left(1 - \xi\Pi^{\zeta-1}\right)^{\frac{1}{\zeta-1}} \\
& G''2\zeta\Pi^{2(\zeta-1)}\left(1 - \xi\Pi^{\zeta-1}\right)^{\frac{1}{\zeta-1}-1} - \xi\zeta(\zeta-2)\Pi^{\zeta-3}\left(1 - \xi\Pi^{\zeta-1}\right)^{\frac{1}{\zeta-1}}.
\end{aligned}
$$

About a zero net inflation steady state, $\Pi = \Delta = 1$ and we have:

$$
\begin{aligned}
F_{\Pi}(1,1) &= \zeta, \\
F_{\Pi\Pi}(1,1) &= \zeta(\zeta-1), \\
G'(1) &= -\xi\zeta\left(1-\xi\right)^{\frac{1}{\zeta-1}} \\
G''2\zeta\left(1-\xi\right)^{\frac{1}{\zeta-1}-1} - \xi\zeta(\zeta-2)\left(1-\xi\right)^{\frac{1}{\zeta-1}}.
\end{aligned}
$$

Hence the terms in $\pi_{H,t}$, $\left(\xi F_{\Pi}(1,1) + (1-\xi)^{\frac{1}{1-\zeta}} G'(1)\Pi\right)\pi_{H,t} = 0$. In other words, about a zero net inflation steady state only second-order terms in inflation affect dispersion. Then, with a little algebra, (D.19) follows from (D.21) and the derivatives above.

Now we complete the quadratic approximation using (D.19):

$$
\sum_{t=0}^{\infty}\beta^t\delta_t = \sum_{\tau=1}^{\infty}\beta^{\tau-1}\delta_{\tau-1} = \beta^{-1}\sum_{t=1}^{\infty}\beta^t\delta_{t-1} = \beta^{-1}(\sum_{t=0}^{\infty}\beta^t\delta_{t-1} - \delta_{-1}). \tag{D.22}
$$

Then assuming that prior to the optimization exercise the economy is at its steady state, $\delta_{-1} = 0$, and using (D.22), we have that

$$
\sum_{t=0}^{\infty}\beta^t\delta_{t-1} = \beta\sum_{t=0}^{\infty}\beta^t\delta_t \Rightarrow \sum_{t=0}^{\infty}\beta^t(\delta_t - \xi\delta_{t-1}) = (1-\xi\beta)\sum_{t=0}^{\infty}\beta^t\delta_t. \tag{D.23}
$$

Hence, from (D.19) and (D.23) up to o(2) we have

$$\sum_{t=0}^{\infty}\beta^t\delta_t = \sum_{t=0}^{\infty}\beta^t\widehat{\delta}_t = \frac{\xi\zeta}{2(1-\beta\xi)(1-\xi)}\sum_{t=0}^{\infty}\beta^t\pi_{H,t}^2. \tag{D.24}$$

We can now write the intertemporal social welfare loss as:

$$\Omega_0 = \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left[-(1-w_C)(1-\lambda)x_t + \pi_{H,t}^2 + \frac{\Psi\,(1+\varphi)}{\varsigma\lambda}x_t^2\right], \tag{D.25}$$

where $\Psi \equiv \frac{(1-\beta\xi)(1-\xi)}{\xi}$. The terms in x_t in (D.25) can be written as $\frac{w_C(1+\varphi)}{2\lambda}(x_t - x_t^{bliss})^2$ where x_t has a *bliss point* $x_t = x_t^{bliss} = \frac{(1-w_C)(1-\lambda)\lambda}{w_C(1+\varphi)}$. This confirms the non-social-optimality of the optimal equitable allocation emphasized in Proposition 3. The bliss point as a function of λ reaches a maximum of $\frac{(1-w_C)}{4w_C(1+\varphi)}$ at $\lambda = \frac{1}{2}$. For typical values $w_C = 0.7$ and $\varphi = 2$ this gives $x_t^{bliss} = 5\%$.

D.2 Proof of Proposition 7

First we introduce the lag operator L and write (4.21) as:

$$\mu_t = \frac{1}{\kappa}\left(\varpi x_t - \Lambda_x + \left(1-\frac{L}{\beta}\right)\nu_t\right). \tag{D.26}$$

Substituting for μ_t into (4.22) we arrive at

$$\pi_{H,t} + \frac{1}{\kappa}(1-L)\left(\varpi x_t - \Lambda_x + \left(1-\frac{L}{\beta}\right)\nu_t\right) - \frac{\Xi w_C}{\beta}L\nu_t = 0. \tag{D.27}$$

Now define:

$$d_t = \pi_{H,t} + \frac{1}{\kappa}(1-L)\left(\varpi x_t - \Lambda_x\right) = \pi_{H,t} + \frac{1}{\kappa}\varpi(x_t - x_{t-1}), \tag{D.28}$$

as a departure from the standard optimal condition (or 'wedge'), where $\pi_{H,t} + \frac{1}{\kappa}\varpi(x_t - x_{t-1}) = 0$ in the case of no penalizing of the interest rate variance when $w_r = 0$. Then (D.27) becomes:

$$d_t = -\frac{1}{\kappa}\left(1 - \left(1 + \frac{1}{\beta} + \frac{\kappa\Xi w_C}{\beta}\right)L + \frac{L^2}{\beta}\right)\nu_t \equiv -\frac{1}{\kappa}F(L)\nu_t. \tag{D.29}$$

Combining (4.23) and (D.29) we arrive at a relationship between the wedge and the nominal interest rate:

$$d_t = \frac{w_r}{\kappa \Xi w_C} F(L) r_t = \frac{w_r}{\kappa \Xi w_C} \left(r_t - \left(1 + \frac{1}{\beta} + \frac{\kappa \Xi w_C}{\beta} \right) r_{t-1} + \frac{1}{\beta} r_{t-2} \right). \tag{D.30}$$

Equation (D.30) defines the optimal policy in terms of a second-order process for the interest rate r_t and a target in the form of a wedge d_t. In the long run it converges to $r_t = -\frac{\frac{\kappa \Xi w_C}{w_r}}{\frac{\kappa \Xi w_C}{\beta}} d_t = -\frac{\beta}{w_r} d_t$. A first-order dynamic approximation to (D.30) that resembles a Taylor-type rule with (super) inertia and has the same long-run property is then:

$$d_t = \frac{w_r}{\kappa \Xi w_C} F(L) r_t = \frac{w_r}{\kappa \Xi w_C} \left(r_t - \left(1 + \frac{\kappa \Xi w_C}{\beta} \right) r_{t-1} \right). \tag{D.31}$$

We now investigate another first-order Taylor-type rule with interest-rate inertia that approximates the optimal rule. The expression $F(L)$ has two roots, one within and one outside the unit circle. It follows that $r_t = \frac{\kappa \Xi w_C}{w_r} F(L)^{-1} d_t$, which potentially gives an interest rate rule responding to past wedges, is not a convergent infinite series, hence ruling out this option. This is a problem of non-invertibility found in the econometrics literature when faced with the existence of a SVAR representation of the RE solution of a macroeconomic DSGE model.

Now define:

$$F(L) = (L - \gamma_1)(L - \gamma_2), \tag{D.32}$$

where $\gamma = \gamma_i, i = 1, 2$ are the roots of $F(L) = 0$. Then the two roots are given by:

$$\gamma = \frac{1 + \frac{1}{\beta} + \frac{\kappa \Xi w_C}{\beta} \pm \sqrt{\left(1 + \frac{1}{\beta} + \frac{\kappa \Xi w_C}{\beta} \right)^2 - \frac{4}{\beta}}}{2}. \tag{D.33}$$

Now consider two cases: (1) $\Xi > 0$; and (2) the IADL case $\Xi < 0$. For case (1), since $\left(1 + \frac{1}{\beta} + \frac{\kappa \Xi w_C}{\beta} \right)^2 - \frac{4}{\beta} = \left(1 - \frac{1}{\beta} \right)^2 + 2 \left(1 + \frac{1}{\beta} \frac{\kappa \Xi w_C}{\beta} \right) + \frac{(\kappa \Xi w_C)^2}{\beta^2}$ both roots are real. Moreover it can be shown that one root say, $\gamma_1 > 1$, and the other, $\gamma_2 < 1$. The existence of the latter root inside the unit circle means $F(L)$ is not invertible. To resolve the problem put

$$F(L)r_t = (L-\gamma_1)(1-\gamma_2)r_t = (L-\gamma_1)(1-\gamma_2 L)\frac{(L-\gamma_2)}{(1-\gamma_2 L)}r_t \equiv (L-\gamma_1)(1-\gamma_2 L)B(L)r_t, \tag{D.34}$$

where

$$B(L) = \frac{L-\gamma_2}{1-\gamma_2 L} \tag{D.35}$$

is a Blaschke factor with the property that $B(L^{-1}) = (B(L))^{-1}$.

Let $r_t^* \equiv B(L)r_t$. Then it is easy to show that $r_t^* = r_t - \mathbb{E}_{t-1}r_t$ is the one-period-ahead prediction error (the 'innovation'). From (D.30) we now can express the rule in terms of r_t^* and past realizations of the wedge $d_{t-i}, i = 0, 1, \cdots$

$$\begin{aligned}
r_t^* &= \frac{\frac{\kappa \Xi w_C}{w_r}}{(L-\gamma_1)(1-\gamma_2 L)}d_t && \text{(D.36)}\\
&= -\frac{\frac{\kappa \Xi w_C}{w_r \gamma_1}}{\gamma_1(1-\frac{L}{\gamma_1})(1-\gamma_2 L)}d_t && \text{(D.37)}\\
&= -\frac{\kappa \Xi w_C}{w_r \gamma_1}\left(1+\frac{L}{\gamma_1}+\left(\frac{L}{\gamma_1}\right)^2+\cdots\right)\left(1+\gamma_2 L+(\gamma_2 L)^2+\cdots\right)d_t && \text{(D.38)}\\
&= -\frac{\kappa \Xi w_C}{w_r \gamma_1}\sum_{i=0}^{\infty}\pi_i d_{t-i}, && \text{(D.39)}
\end{aligned}$$

where because $\frac{1}{\gamma_1} < 1$ and $\gamma_2 < 1$ the series in the lag operator L are convergent.

Note that from (D.35) we have

$$(1-\gamma_2 L)r_t^* = r_t^* - \gamma_2 r_{t-1}^* = (L-\gamma_2)r_t = r_{t-1} - \gamma_2 r_t. \tag{D.40}$$

It follows that we can write the interest rate rule as:

$$r_t - \frac{1}{\gamma_2} r_{t-1} = \frac{\kappa \Xi w_C}{w_r \gamma_1} \sum_{i=0}^{\infty} \tilde{\pi}_i d_{t-i}, \tag{D.41}$$

which since $\frac{1}{\gamma_2} > 1$ is a super-inertial rule in past wedges. The rule can be implemented using a finite order approximation to the infinite series. For example a first order approximation gives

$$r_t = \frac{1}{\gamma_2} + \frac{\kappa \Xi w_C}{w_r \gamma_1} \left[d_t + \left(\frac{1}{\gamma_1} + \gamma_2 \right) d_{t-1} \right]. \tag{D.42}$$

All this assumes real roots for (D.33) which holds for the standard case without IADL, but not for the IADL case. In this case only the approximation (D.31) is available, which can be written as the rule (4.26). This completes the proof. □

D.3 Implementation of optimal policy

We can relate the approximation of the optimal rule of equation (4.26) to the implementation in section 3. First note, the approximation to the optimal rule can be written as

$$r_t = \left(1 + \frac{\kappa \Xi w_C}{\beta} \right) r_{t-1} + \frac{\kappa \Xi w_C}{w_r} \pi_{H,t} + \frac{\kappa \Xi w_C}{w_r} \frac{\varpi}{\kappa} (x_t - x_{t-1}). \tag{D.43}$$

For determinacy analysis, we can set all shocks to zero and we have $x_t = y_t$. We substitute in the NKPC (4.13) then (C.13) to give

$$\begin{aligned} r_t = & \left(\frac{1 + \frac{\kappa \Xi w_C}{\beta}}{1 + \frac{\beta \kappa \Xi (1 - w_C)}{w_r}} \right) r_{t-1} + \frac{\frac{\kappa \Xi \beta}{w_r}}{1 + \frac{\beta \kappa \Xi (1 - w_C)}{w_r}} \pi_{t+1} \\ & + \frac{\frac{\Xi w_C}{w_r} \kappa^2}{1 + \frac{\beta \kappa \Xi (1 - w_C)}{w_r}} y_t + \frac{\frac{\Xi w_C}{w_r} \varpi}{1 + \frac{\beta \kappa \Xi (1 - w_C)}{w_r}} (y_t - y_{t-1}). \end{aligned} \tag{D.44}$$

Note that using (C.11) and (C.24), and combining with (4.13) then (C.13), we can write output growth as:

$$y_t - y_{t-1} = -\frac{\Xi}{\sigma} \left(w_C r_{t-1} - w_C \beta \frac{1}{w_C} \pi_{t+1} + w_C \beta \frac{1 - w_C}{w_C} r_t - w_C \kappa y_t \right). \tag{D.45}$$

This can be substituted into the rule to give:

$$
\begin{aligned}
r_t = \left(\frac{1 + \left(\frac{\kappa}{\beta} - \frac{\Xi}{\sigma} \frac{\varpi w_C}{w_r} \right) \Xi w_C}{1 + \frac{\beta \Xi (1 - w_C)}{w_r} \left(\kappa + \frac{\Xi w_C \varpi}{\sigma} \right)} \right) r_{t-1} \\
+ \left(\frac{\kappa + \frac{\Xi}{\sigma} w_C \varpi}{1 + \frac{\beta \Xi (1 - w_C)}{w_r} \left(\kappa + \frac{\Xi w_C \varpi}{\sigma} \right)} \frac{\Xi}{w_r} \beta \right) \pi_{t+1} \\
+ \left(\frac{\kappa + \frac{\Xi}{\sigma} w_C \varpi}{1 + \frac{\beta \Xi (1 - w_C)}{w_r} \left(\kappa + \frac{\Xi w_C \varpi}{\sigma} \right)} \frac{\Xi}{w_r} w_C \kappa^2 \right) y_t,
\end{aligned}
\quad \text{(D.46)}
$$

or

$$
r_t = \rho r_{t-1} + \theta_\pi \pi_{t+1} + \theta_y y_t. \quad \text{(D.47)}
$$

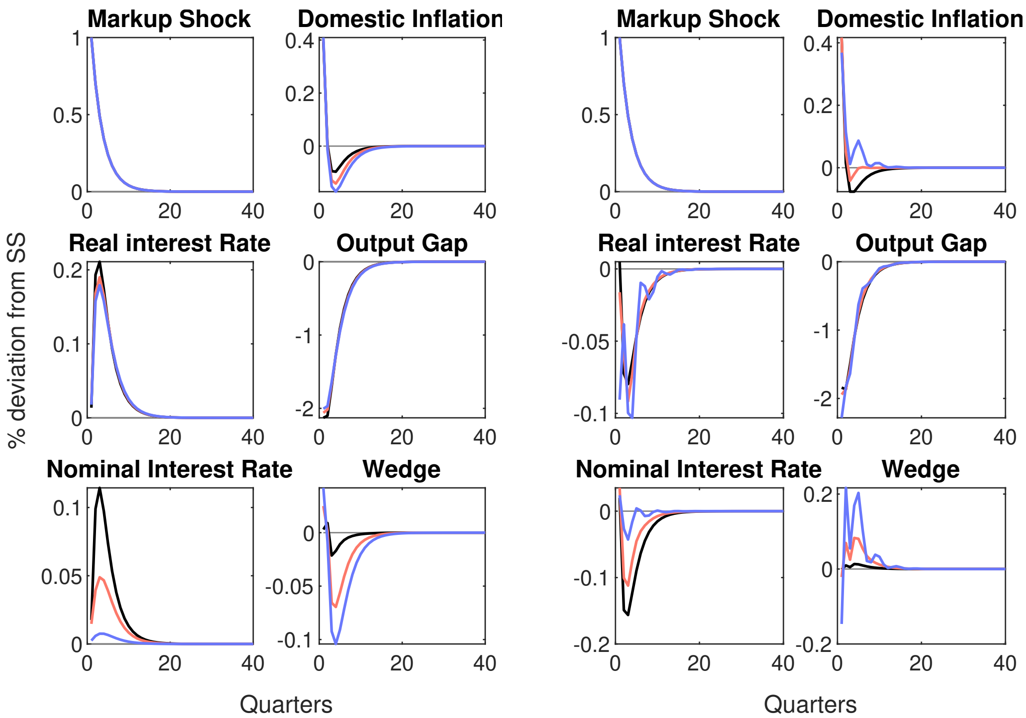
Finally, note that $\theta_y = \frac{w_C \kappa^2}{\beta} \theta_\pi$ and that κ is between 0.25 and 0.4 in the baseline parameterizations, θ_π is approximately 6–16 times larger than θ_y, and so we can approximate the rule with:

$$
r_t = \rho r_{t-1} + \theta_\pi \pi_{t+1}. \quad \text{(D.48)}
$$

D.4 Optimal policy numerical simulations

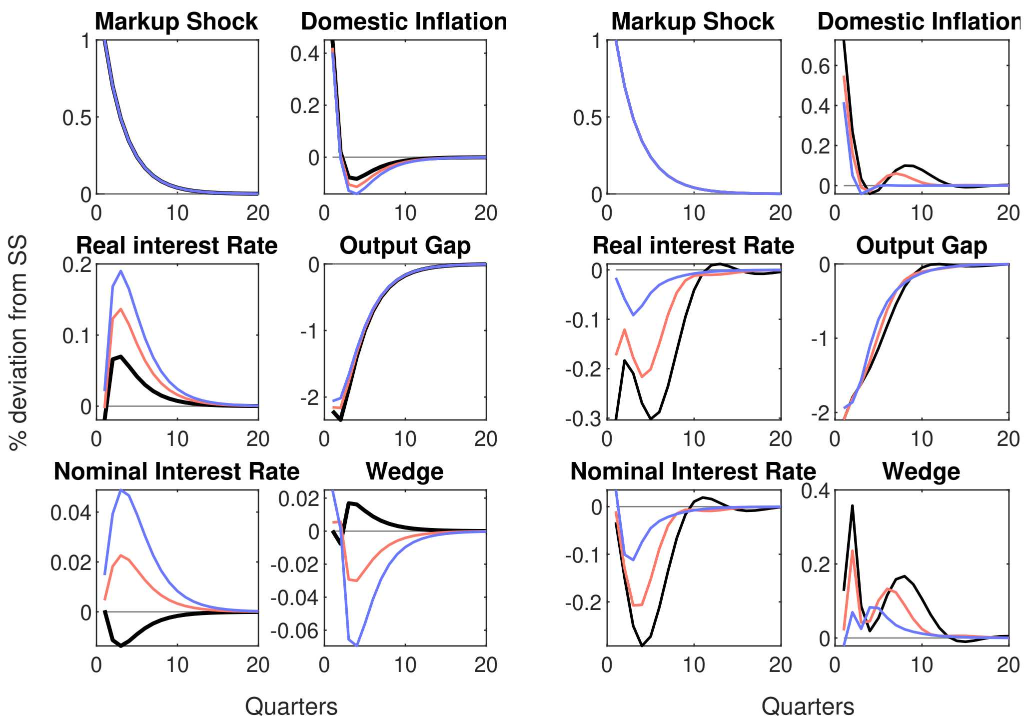
Figures 20 and 21 present simulations in response to an AR(1) mark-up shock for the optimal policy given in equation (4.25). We set the persistence parameter at $\rho_u = 0.7$ and for the remaining parameters we use the restricted parameterization given in Section 4.2. Figure 20 displays the effect of higher interest-rate stabilization brought about by increasing the penalty parameter w_r for the cases of $\lambda > \lambda^*$ (panel (a)) and $\lambda < \lambda^*$ (panel (b)), where λ^* is the IADL threshold given by (4.18), which equals $\lambda^* = 0.546$ with $w_C = 0.6$ and $\varphi = 2$. This nearly exactly resembles the simulations using the the first-order Taylor-type rule shown in Figure 9 but without the presence of a small degree of oscillations in the IADL case.

Figure 21 compares differing degrees of trade openness under the optimal policy. Again, this is similar to the outcome under the approximation to the optimal policy but this time the oscillations are seen to be more pronounced, especially in the closed economy. This is because in the IADL case, monetary policy responds less to inflation and output and has a higher degree of policy inertia.



(a) SADL ($\lambda = 0.7 > \lambda^*$) (b) IADL ($\lambda = 0.5 < \lambda^*$)

Figure 20: Optimal policy and variance penalty for $w_r = 0.1$ (black line), $w_r = 1$ (red line) and $w_r = 10$ (blue line). Parameter values are $\lambda = 0.5, 0.7$, $\Psi = 0.086$, $\varphi = 2$, $\beta = 0.99$, $\sigma = \mu_C = 1$, $w_C = 0.6$.



(a) SADL ($\lambda = 0.7 > \lambda^*$) (b) IADL ($\lambda = 0.5 < \lambda^*$)

Figure 21: Optimal policy and openness for $w_C = 1$ (black line), $w_C = 0.8$ (red line) and $w_C = 0.6$ (blue line). Parameter values are $\lambda = 0.5, 0.7$, $\Psi = 0.086$, $\varphi = 2$, $\beta = 0.99$, and $\sigma = \mu_C = w_r = 1$.